First-Best and Second-Best Regulation of Solid Waste under Imperfect Competition in a Durable Good Industry

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Abstract. Under the assumption of imperfect competition in a durable good industry, the present paper investigates the efficient regulation of solid waste which causes environmental damage at the end of the product's life. It turns out that the second-best waste tax falls short of the marginal environmental damage if the producers rent their products but may also exceed the marginal damage if the producers sell their products. If in the sales case the industry is regulated with waste taxes and stock subsudies then the first-best tax-subsidy scheme also contains a waste tax which deviates from the marginal damage, in general. Under monopoly this tax unambiguously exceeds the marginal damage. Furthermore, the analysis provides a further reason why the Swan independence result generally doesn't hold in rental markets.

Keywords: durability, first- and second-best taxation, monopoly, oligopoly, solid waste

JEL classification: L12, L13, H21, Q28

1. Introduction

Since the paper of Buchanan (1969) the relationship between environmental policy and market structure has become an issue of increasing interest (e.g. Carraro *et al.* (1996)). A great part of the research effort is devoted to the emission tax which maximizes social welfare in a polluting Cournot industry provided it is the only instrument available. Intuitively, the rate of this second-best emission tax falls short of the Pigouvian level (underinternalization, i.e. the tax rate is smaller than the marginal environmental damage) since it has to account simultaneously for the environmental externality and the market imperfection which both tend to divert the industry emission from its efficient level but in opposite directions. Under several restrictive assumptions underinternalization has been proven for monopoly (Misolek (1980), Barnett (1980)) as well as for oligopoly (Ebert (1992)). A second-best emission tax which exceeds the marginal damage (overinternalization) can't be ruled out, however, if the producers have at their disposal a rather general abatement technolgy (Barnett (1980), Ebert (1992)), if the number of firms is endogenous (Katsoulacos/Xepapadeas (1995), Requate (1997)) or if the emission in a durable good industry is a rather general function of durability and output (Goering/Boyce (1999), Runkel (1999b)).[1]

Unfortunately, a closer look at the overinternalization cases suggests that despite their theoretical elegance these cases seem to have counterintu-

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itive implications. More specifically, in the models of Barnett (1980), Ebert (1992), Katsoulacos/Xepapadeas (1995) and Requate (1997) the second-best emission tax exceeds the marginal damage only if the equilibrium output or the equilibrium emission of the individual firm *increases* as the emission tax increases. Economic intuition certainly predicts the opposite effect to be more relevant. A similar argument applies to the durable good model of Goering/Boyce (1999) and Runkel (1999b). In their model overinternalization requires that the equilibrium durability or the equilibrium output *increases* as the emission tax increases. However, both variables are positive correlated with the equilibrium emission which unambiguously *decreases* as the tax increases. Moreover, they only consider the case in which the durables are rented. Apart from some exceptions like copiers, however, the major part of durable goods is not rented but sold by their producers. Of course, the above arguments are no substitute for hard empirical evidence and hence the relevance of the overinternalization cases previously derived in literature remains a debatable issue for the time being.

Anyhow, the main purpose of the present paper is to provide further theoretical support for overinternalization in the second-best optimum *without* the need of those conditions used in previous literature that strain economic intuition. To this end, a two-period model of a durable good industry under imperfect competition is investigated. The model differs from previous models on product durability (e.g. Bulow (1986), Goering (1992)) in explicitly recognizing the environmental damage caused by the solid waste at the end of the product's life. The distinctive feature with respect to the second-best taxation models refered to above is that the model allows to investigate both the rental case and the more realistic case in which the producers sell their products. In the rental case it turns out that the waste (emission) tax is negatively correlated with the equilibrium amount of waste and the equilibrium stock of the durable and that the second-best waste tax falls always short of the marginal environmental damage. In the sales case, however, overinternalization may be second-best optimal and that *without* the requirement of atypical reactions on tax changes. As will be argued, the rationale of this result lies in an additional distortion only inherent in sales market for durable goods, namely a durability which doesn't miminize the social cost of providing a given service level even if the environmental externality is completely internalized. Thus, in sales markets an increase in the emission tax may have the additional benefit of mitigating this distortion and hence overinternalization may be second-best optimal.

In addition to the propositions on second-best taxation the present paper offers two other important results. Firstly, by introducing further policy instruments it is possible to investigate tax-subsidy schemes which render the industry equilibrium Pareto efficient (first-best). For the rental case, a first-best tax-subsidy scheme is shown to consist of a Pigouvian waste tax which completely internalizes the environmental damage and a subsidy on the stock of the durable which corrects the market imperfection. Suppose, however, the firms sell their products and the regulator uses waste taxes and stock subsidies. Then, similar as in case of second-best taxation, the first-best tax-subsidy scheme contains a waste tax which deviates from the marginal environmental damage, in general. Under monopoly this waste tax unambiguously exceeds the marginal damage. Secondly, the present analysis challenges the so-called Swan independence result: From the industrial organization literature it is known that in sales markets durability generally depends on the number of producers (Bulow (1986)) whereas in rental markets it doesn't (Swan (1970), Goering (1992)). However, the present paper shows durability to depend on market structure in rental markets, too, if it is regulated by means of a tax or a subsidy.

The paper proceeds as follows. Section 2 presents the model and derives the first-best social optimum. In section 3, conditions for a rental and a sales equilibrium of the durable good industry are determined. Section 4 investigates the first-best regulation of the industry, and section 5 establishs the main results of the paper with respect to second-best emission taxation. Section 6 concludes.

2. Assumptions and Social Optimum

Consider a two-period model of a durable good industry which consists of a fixed number of firms, $n \geq 1$. y_{ti} is the production rate of firm $i \,\epsilon\, \{1, \ldots, n\}$ in period $t = 1, 2$. $\phi_i \,\epsilon\, [0, 1]$ represents i's average product durability, i.e. the fraction of i's first-period production which is still available for use in the second period. If the firms rent their products to the consumers then $c_{1i} := y_{1i}$ and $c_{2i} := y_{2i} + \phi_i y_{1i}$ define i's stock of the durable in period 1 and 2, respectively. If the firms sell their products then c_{ti} is interpreted as i's 'effective market share' in t because in this case the stock is held by the consumers. Firm i's products cause the amount of solid waste $w_{1i} = (1 - \phi_i)c_{1i}$ in period 1 and $w_{2i} = c_{2i}$ in period 2. Hence, an increase in durability c.p. leads to a decrease in the (average) amount of solid waste. For the industry variables the following notations are introduced: $Y_t := \sum_j y_{tj}$, $C_t := \sum_j c_{tj}$ and $W_t := \sum_j w_{tj}$ are the industry production in t, the industry stock in t and the industry amount of waste in t, respectively. $\bar{\phi} Y_1 := \sum_j \phi_j y_{1j}$ stands for the remaining first-period production of the industry. $C_t^{-i} := \sum_{j \neq i} c_{tj}$ denotes the durable stock of i's competitors in t.

The cost side of the economy consists of two components. Firstly, each firm faces the same production cost function which is linear in output (constant returns to scale) and strictly convex in durability (decreasing returns to durability). Firm i's production costs are denoted by $K(\phi_i)y_{1i}$

with $K' > 0$ and $K'' > 0$ in period 1 and $K(0)y_{2i} = ky_{2i}$ with $k > 0$ in period 2. Secondly, the solid waste W_t of the durable good is assumed to cause environmental damage. This damage results from waste transport, disposal, incineration and landfilling. It is evaluated by the damage function $D(W_t)$ with $D' > 0$ and $D'' \geq 0$, i.e. a greater amount of solid waste causes a greater environmental damage at non-decreasing rates.

The demand function $P(C_t)$ with $P' < 0$ describes the demand side of the economy. $P(C_t)$ is the rental price of the durable good in period t as a function of the total industry stock. For $i = 1, \ldots, n$ assume

$$P'(C_t) + c_{ti}P''(C_t) < 0 \qquad \text{for all } c_{ti}, C_t, \qquad t = 1, 2, \tag{1}$$

$$P'(Y_2 + \bar{\phi}Y_1) + y_{2i}P''(Y_2 + \bar{\phi}Y_1) < 0 \qquad \text{for all } y_{2i}, Y_2. \tag{2}$$

These are the Hahn (1962) conditions applied to a two-period durable good oligopoly. (1) states that the marginal rental revenue $P(\cdot) + c_{ti}P'(\cdot)$ of firm i in t decreases with the industry stock of the durable.[2] (2) constitutes the analogous property of the marginal sales revenue $P(\cdot) + y_{2i}P'(\cdot)$ in period 2. In the (one-period) nondurable good oligopoly there is much motivation for the Hahn conditions because together with $P' < 0$ they a) ensure uniqueness and stability of the Cournot equilibrium (Friedman (1982), theorem 1; Furth (1986), theorem 3.3), b) ensure downward-sloping reaction functions for all firms (Dixit (1986), pp. 118), and c) exclude some perverse effects in the comparative statics (McElroy (1992)). As will be shown in the sections 3 and 4, the assumptions (1) and (2) have similarly useful implications in the present durable good oligopoly although this is a two-period model.

To characterize the socially optimal outcome, assume the social planner seeks a symmetric solution $c_{ti} = c_t$ $(t = 1, 2)$ and $\phi_i = \phi$ for all i. Thus, she solves the problem of

$$\max_{c_1, c_2, \phi} V(c_1, c_2, \phi) := S(nc_1) - K(\phi)nc_1 - D[(1-\phi)nc_1] + \rho\Big\{S(nc_2) - kn(c_2 - \phi c_1) - D(nc_2)\Big\}. \tag{3}$$

In both periods the social welfare equals the consumer benefit $S(C_t) = \int_0^{C_t} P(x)dx$ less production and environmental costs. $\rho \,\epsilon\, [0, 1]$ denotes the social discount factor. The first-order conditions for the welfare maximum are listed in column 1, row 1 to 3 of table I. These conditions are the 'marginal benefit equal to marginal cost' requirements: For a marginal increase in the stock of the durable good the LHS of the corresponding condition gives the marginal benefit which equals the marginal consumer benefit and the RHS gives the marginal costs which consist of the production and the environmental costs (for c_1 use k from the condition of c_2). For a marginal decrease in product durability the LHS of the pertinent condition equals the benefit, i.e. the saved production cost, whereas the RHS equals the marginal costs,

Table I. Equilibrium Conditions, First-Best and Second-Best Policies

column		1	2
row	variable	social optimum	oligopoly equilibrium ($\delta = 0$: rental equilibrium; $\delta = 1$: sales equilibrium)
Efficiency and Equlibrium Conditions:			
1	c_1	$P_1 + \rho k\phi = K + (1-\phi)D_1'$	$P_1 + c_1 P_1' + \rho k\phi = K + (1-\phi)\tau_{w_1} + \tau_{c_1} + \tau_{y_1} - \rho\phi\tau_{y_2} - \delta\rho\phi z$
2	c_2	$P_2 = k + D_2'$	$P_2 + (c_2 - \delta\phi c_1)P_2' = k + \tau_{w_2} + \tau_{c_2} + \tau_{y_2}$
3	ϕ	$K' = \rho k + D_1'$	$K' = \rho k + \tau_{w_1} + \rho\tau_{y_2} - \tau_\phi/c_1 + \delta\rho z$
First-Best Policies:			
4	c_1	$(1-\phi)\tau_{w_1} + \tau_{c_1} + \tau_{y_1} - \rho\phi\tau_{y_2} = (1-\phi)D_1' + c_1 P_1' + \delta\rho\phi z$	
5	c_2	$\tau_{w_2} + \tau_{c_2} + \tau_{y_2} = D_2' + (c_2 - \delta\phi c_1)P_2'$	
6	ϕ	$\tau_{w_1} + \rho\tau_{y_2} - \tau_\phi/c_1 = D_1' - \delta\rho z$	
Second-Best Waste Taxes:			
7	τ_{w_1}	$\tau_{w_1} = D_1' + c_1 P_1' \dfrac{\varepsilon^1_{c_1}\varepsilon^2_{c_2} - \varepsilon^2_{c_1}\varepsilon^1_{c_2}}{(1-\phi)(\varepsilon^1_{w_1}\varepsilon^2_{c_2} - \varepsilon^2_{w_1}\varepsilon^1_{c_2})} + \delta\rho\phi z \dfrac{(\varepsilon^1_{c_1} + \varepsilon^1_\phi)\varepsilon^2_{c_2} - (\varepsilon^2_{c_1} + \varepsilon^2_\phi)\varepsilon^1_{c_2}}{(1-\phi)(\varepsilon^1_{w_1}\varepsilon^2_{c_2} - \varepsilon^2_{w_1}\varepsilon^1_{c_2})}$	
8	τ_{w_2}	$\tau_{w_2} = D_2' + (c_2 - \delta\phi c_1)P_2' + c_1 P_1' \dfrac{c_1(\varepsilon^2_{c_1}\varepsilon^1_{w_1} - \varepsilon^1_{c_1}\varepsilon^2_{w_1})}{\rho c_2(\varepsilon^2_{c_2}\varepsilon^1_{w_1} - \varepsilon^1_{c_2}\varepsilon^2_{w_1})} + \delta\rho\phi z \dfrac{c_1[(\varepsilon^2_{c_1} + \varepsilon^2_\phi)\varepsilon^1_{w_1} - (\varepsilon^1_{c_1} + \varepsilon^1_\phi)\varepsilon^2_{w_1}]}{\rho c_2(\varepsilon^2_{c_2}\varepsilon^1_{w_1} - \varepsilon^1_{c_2}\varepsilon^2_{w_1})}$	

Definitions/Notation: $P_t := P(nc_t),\quad P_t' := P'(nc_t),\quad D_t' := D'(nw_t),\quad \varepsilon^t_x := (\partial x/\partial\tau_{w_t})\cdot\tau_{w_t}/x,\quad t = 1,2,\quad x \in \{c_1, c_2, \phi, w_1\}$

$$z := \phi c_1 P_2' \frac{\partial c_2}{\partial\phi c_1} + c_2 P_2' \frac{\partial C_2^-}{\partial\phi c_1} \quad \text{with} \quad \frac{\partial c_2}{\partial\phi c_1} = \frac{nP_2' + (n-1)(c_2 - \phi c_1)P_2''}{(n+1)P_2' + n(c_2 - \phi c_1)P_2''} > 0, \quad \frac{\partial C_2^-}{\partial\phi c_1} = -\frac{(n-1)P_2' + (n-1)(c_2 - \phi c_1)P_2''}{(n+1)P_2' + n(c_2 - \phi c_1)P_2''} \leq 0$$

i.e. the additional production cost to maintain the stock of the durable good in period 2 and the additional environmental cost of an increasing amount of solid waste in period 1.

Table II contains some comparative dynamic results that are obtained by totally differentiating the first-order conditions for the social optimum after two shift parameters κ_P, κ_D with $\partial P/\partial\kappa_P$, $\partial D'/\partial\kappa_D > 0$ have been introduced. With respect to κ_P, κ_D and ρ the results are the same as for

Table II. Comparative dynamic results for the social optimum

	∂c_1	∂c_2	$\partial\phi$	∂nc_1	∂nc_2	∂nw_1	∂nw_2
$\partial\kappa_P$	>0	>0	>0	>0	>0	>0	>0
$\partial\kappa_D$	<0	<0	>0	<0	<0	<0	<0
$\partial\rho$	>0	$=0$	>0	>0	$=0$	$\gtrless 0$	$=0$
∂n	<0	<0	$=0$	$=0$	$=0$	$=0$	$=0$

the case with an infinite number of firms investigated by Runkel (1999a): Enlarging the market requires an increase in the efficient durability in order to partially compensate for the effect the increased stock of the durable exerts on the amount of waste. An increasing marginal damage brings about an increase in the efficient durability and a decrease in the efficient stock of the durable good in order to reduce the amount of waste since every scrapped unit causes a greater damage. A decreasing discount factor forces future generations to produce a greater part of their durable stock themselves, i.e. the second-period production increases since the second-period stock remains constant whereas the remaining first-period stock declines due to a decrease in the durability and the first-period stock. A new and particularly interesting result in table II is that an increase in the number of firms has neither an effect on the efficient durability nor on the efficient values of the aggregate variables of the model. The only effect is that the individual quantities decrease since unchanged industry quantities are distributed among a greater number of firms.

3. Rental vs. Sales Equilibrium

Now suppose that the durable good is supplied by profit-maximizing firms which either rent or sell their goods. Incorporated into the models to be studied in this section are various taxes and subsidies even though the welfare economic rationale of such policy instruments hasn't been demonstrated until now. The analysis in this section should be viewed as a descriptive analysis of a regulated durable good industry disregarding efficiency implications. This view will prove to be useful for investigating the validity of Swan's independence result in proposition 1. The normative analysis is postponed to the sections 4 and 5 where certain taxes and subsidies are shown to be capable of correcting market failure under laissez-faire.

In the *rental* case a firm remains the owner of the units it produces. Instead of selling the good the producer sells the services of the good. Firm i's rental profits in period 1 and 2 are, respectively,

$$\Pi_{1i}^r = c_{1i}P(C_1) - c_{1i}[K(\phi_i) + (1-\phi_i)\tau_{w_1} + \tau_{c_1} + \tau_{y_1}] - \phi_i\tau_\phi, \tag{4}$$

$$\Pi_{2i}^r = c_{2i}P(C_2) - (c_{2i} - \phi_i c_{1i})(k + \tau_{y_2}) - c_{2i}(\tau_{w_2} + \tau_{c_2}). \tag{5}$$

The present value of firm i's rental profit is

$$\Pi_i^r = \Pi_{1i}^r + \rho\Pi_{2i}^r. \tag{6}$$

In every period the profit equals the rental revenue less production cost and tax payments. The rental profit depends on several tax rates which are taken as given by the firms: The output in period t is taxed at the rate τ_{y_t}. τ_{c_t} denotes the tax rate on the firm's stock of the durable good in t. τ_{w_t} is the waste tax rate in t. The durability is taxed at the rate τ_ϕ. If a tax rate is negative then it is a subsidy. The regulator announces all tax rates right at the beginning of period 1.

The decision problem of a single firm is to choose the stock of the durable good in both periods and the durability such that its profit is maximized. In general, the intertemporal dimension of this decision requires to distinguish between open-loop and closed-loop information structures (Fudenberg/Tirole (1994), pp. 130). In rental markets, however, both information structures yield the same market equilibrium: If firm i plays open-loop strategies then it takes as given the entire time paths chosen by its competitors, and thus it assumes the rivals' future actions to be independent of its own current actions. Formally, this means that i maximizes (6) with respect to c_{1i}, ϕ_i and c_{2i} and takes as given c_{tj} for all $j \neq i$, $t = 1, 2$. The first-order conditions $\partial\Pi_i^r/\partial x = 0$ for every $x \in \{c_{1i}, c_{2i}, \phi_i\}$ determine i's best response to the actions of its competitors. The conditions for a symmetric open-loop rental equilibrium are listed in the column 2, row 1 to 3 of table I (set the dummy variable δ equal to zero). If firm i uses closed-loop strategies then it takes into account that its current actions influence the rivals' future actions and that it has to solve the profit maximization recursively. Hence, in period 2 it maximizes (5) with respect to c_{2i} for given c_{1i} and ϕ_i. This is done by every firm and the pertinent first-order conditions

$$P(C_2) + c_{2i}P'(C_2) = k + \tau_{w_2} + \tau_{c_2} + \tau_{y_2}, \qquad i = 1, \ldots, n \tag{7}$$

define a unique and stable Cournot equilibrium of the rental subgame in period 2 due to the Hahn conditions in (1).[3] In period 1 firm i maximizes

(6) over c_{1i} and ϕ_i again, but now recognizes that firm j's stock c_{2j} in 2 is generally a function of the remaining first-period stocks: $c_{2j}(\boldsymbol{\phi}\mathbf{c_1})$ with $\boldsymbol{\phi}\mathbf{c_1} := (\phi_1 c_{11}, \ldots, \phi_n c_{1n})$. In our rental game, however, (7) obviously implies $\partial c_{2j}/\partial \phi_i c_{1i} = 0$ for all $i, j = 1, \ldots, n$, i.e. the first-period decisions don't influence the second-period decisions. This result coincides with the assumption under the open-loop information structure. Hence, the open- and closed-loop rental equilibrium are the same. Both are subgame perfect and both are characterized by the above-mentioned conditions in table I.

Now, turn to the *sales* case. The sales price of a first-period unit of output is represented by the implicit rentals $P(Y_1) + \rho\phi_i P(Y_2 + \bar{\phi}Y_1)$ from both periods whereas the sales price of a second-period unit equals the rental $P(Y_2 + \bar{\phi}Y_1)$ from period 2 only. However, if the firms sell their output then the property rights of the products pass over to the consumers, and the consumers typically have to pay for the stock and the waste taxes. Thus, the consumers adjust the price they are willing to pay by these taxes, and firm i's sales profits in 1 and 2 become

$$\begin{aligned} \Pi_{1i}^s \;&=\; y_{1i}\Big\{P(Y_1) - (1-\phi_i)\tau_{w_1} - \tau_{c_1} + \rho\phi_i[P(Y_2 + \bar{\phi}Y_1) - \tau_{w_2} - \tau_{c_2}]\Big\} \\ &\qquad - y_{1i}[K(\phi_i) + \tau_{y_1}] - \phi_i\tau_\phi, \qquad (8) \\ \Pi_{2i}^s \;&=\; y_{2i}[P(Y_2 + \bar{\phi}Y_1) - \tau_{w_2} - \tau_{c_2}] - y_{2i}(k + \tau_{y_2}). \qquad (9) \end{aligned}$$

Summing (8) and (9) and using the definition of c_{ti} reveals the present value of the sales profits $\Pi_i^s = \Pi_{1i}^s + \rho\Pi_{2i}^s$ to be equal to the present value of the rental profits defined in (6). This doesn't imply, however, that the equilibrium conditions are the same as in the rental case since due to the so-called Coase conjecture (Coase (1972), Gul *et al.* (1986)) every durable good seller faces a commitment problem: In the second period the selling firm has an incentive to supply more durable goods than it in the first period has announced to do since the capital loss on the second-period stock (the reduction in the value of the second-period stock due to the additional supply of the firm) is not born by the seller but by the consumers. Rational consumers, however, anticipate the second-period behaviour of the selling firm and adjust the price they are willing to pay in the first period. Thus, the commitment problem of the selling firm is that it can't credibly promise first-period buyers to account for their second-period capital loss.

Game-theoretically, the commitment problem is closely related to the distinction between open- and closed-loop information structures. If the selling firm plays open-loop strategies then it precommits right at the beginning of the game to its actions in both periods. The commitment problem implies, however, that the selling firm doesn't possess this commitment ability. Thus, in sales markets for durable goods the open-loop information structure isn't an appropriate assumption. In contrast, closed-loop strategies don't require precommitment of the selling firm and hence are suitable to derive an equilibrium of the sales market as follows: In period 2 firm i maximizes Π^s_{2i} from (9) with respect to y_{2i} taking as given the decisions in period 1. The first-order conditions for the n firms read

$$P(Y_2 + \bar{\phi} Y_1) + y_{2i} P'(Y_2 + \bar{\phi} Y_1) = k + \tau_{w_2} + \tau_{c_2} + \tau_{y_2}, \quad i = 1, \ldots, n. \tag{10}$$

When combined with the Hahn conditions (2) this system of equations implicitly determines a unique and stable Cournot equlibrium of the sales subgame in 2 where the equilibrium production rates are functions of the remaining first-period stocks: $y_{2j} = y_{2j}(\boldsymbol{\phi}\mathbf{y_1})$ $(j = 1, \ldots, n)$. By applying the comparative static methods of Dixit (1986), p. 120 to (10) we obtain[4]

$$\frac{\partial y_{2j}}{\partial \phi_i y_{1i}} = -\frac{P_2' + y_{2j} P_2''}{(n+1)P_2' + Y_2 P_2''} \, \epsilon \,]-1, 0[, \quad i, j = 1, \ldots, n \tag{11}$$

or if the model is transformed by using the definition of c_{ti} $(t = 1, 2)$:

$$\frac{\partial c_{2i}}{\partial \phi_i c_{1i}} = \frac{n P_2' + (Y_2 - y_{2i}) P_2''}{(n+1)P_2' + Y_2 P_2''} \, \epsilon \,]0, 1[, \quad i = 1, \ldots, n, \tag{12}$$

$$\frac{\partial c_{2j}}{\partial \phi_i c_{1i}} = -\frac{P_2' + y_{2j} P_2''}{(n+1)P_2' + Y_2 P_2''} \, \epsilon \,]-1, 0[, \quad i, j = 1, \ldots, n; \; i \neq j \tag{13}$$

and

$$\frac{\partial C_2^{-i}}{\partial \phi_i c_{1i}} = -\frac{(n-1)P_2' + (Y_2 - y_{2i}) P_2''}{(n+1)P_2' + Y_2 P_2''} \, \epsilon \,]-1, 0[, \quad i = 1, \ldots, n \tag{14}$$

where the signs of the partial derivatives result from (2) and $P_t' < 0$. In contrast to rental markets, in sales markets product durability and the first-period production play a strategic role: If i increases either of these variables in 1 then in 2 this c.p. reduces all production rates and the effective market share of i's competitors (see (11), (13) and (14)) but raises i's effective market share (see (12)). This strategic incentive is recognized in the first-period decision: In period 1 firm i chooses y_{1i} and ϕ_i in order to maximize Π^s_i subject to (10), i.e. via (11) it takes into consideration the influence its first-period production and durability exerts on the future production rates. The solution to this first-period maximization[5] together with (10) and the assumption of a symmetric equilibrium is listed in column 2 of table I where δ is now set equal to unity.

Summing up, the industry equilibrium is characterized by the conditions of column 2, row 1 to 3 of table I. These conditions are a straightforward generalization of the results of Bulow (1986) and Goering (1992) to the case in which several taxes regulate the industry: For the symmetric rental equilibrium $(\delta = 0)$ the first two conditions say that the marginal revenue of the stock in one period equals the marginal production costs plus tax payments of that stock. Due to the third equation the saved production costs of a decreasing durability are offset by the additional cost of maintaining the stock in period 2 and the additional tax payments. For the symmetric sales equilibrium ($\delta = 1$) the resulting equations have almost the same interpretation as in the rental case except for the additional terms containing z. z highlights the two special features of product durability in sales markets: The first term in z, namely $\phi c_1 P_2' \cdot (\partial c_2 / \partial \phi c_1) < 0$, is called planned obsolescence since it provides the firms with the incentive to stimulate the second-period sales by *reducing* the product durability below its value in the rental case (see the last of the three equilibrium conditions). Under oligopoly ($n > 1$) the second term in z, namely $c_2 P_2' \cdot (\partial C_2^- / \partial \phi c_1) > 0$, reflects the incentive for the individual firm to decrease the rivals' future market share and to increase its own share by *raising* durability beyond its value in the rental case. Consequently, this expression is termed strategic effect of product durability. Under oligopoly the sign of z is ambiguous whereas under monopoly ($n = 1$) the strategic effect vanishs and z is negative. Since the obsolescence effect and the strategic effect influence both the durability and the stock of the durable good they will also affect the amount of solid waste. Hence, they will have crucial implications for the environmental regulation of the industry.

Before proceeding with the analysis of these implications a short remark on Swan's independence theorem is in order since in contrast to the most previous works on this issue the present model incorporates several taxes. Of course, Bulow's (1986) result that in the sales case product durability depends on the number of firms also carries over to the more general model with taxes. However, Swan's (1970) and Goering's (1992) results that in rental markets product durability doesn't depend on market structure is challenged by the present model. To see this, consider the equilibrium conditions in column 2 of table I and ignore all their normative implications, i.e. suppose, temporarily, that the regulator levies the same tax rates for every number of firms without aiming at the efficient regulation of the industry. Focusing on the rental case ($\delta = 0$) and totally differentiating the equilibrium conditions yields

$$\frac{\partial \phi^r}{\partial n} = \frac{1}{|J^r|} \left(P_1' + c_1^r P_1'' \right) \left[(n+1) P_2' + n c_2^r P_2'' \right] \frac{-\tau_\phi}{c_1^r} \tag{15}$$

where the superscript r identifies the rental case. The Jacobian determinant $|J^r| = [(n+1)P_2' + n c_2^r P_2''] \{K''[(n+1)P_1' + n c_1^r P_1''] + \tau_\phi^2 / c_1^{r3}\}$ is positive due to (1) and the second-order conditions for the profit maximum of a renting firm. This together with (1) ensures the three first terms in (15) to be positive. Hence, the sign of $\partial \phi^r / \partial n$ equals the sign of $-\tau_\phi$ and we have proven

PROPOSITION 1. (*Durability and Market Structure*) *In the rental equilibrium, the durability ϕ^r is independent of n if and only if $\tau_\phi = 0$.*

The intuition of this result becomes clear by setting δ and all tax rates except τ_ϕ equal to zero in the equilibrium conditions of table I. If τ_ϕ is negative, i.e. a subsidy, then the marginal cost of increasing durability consists of the additional production cost K', and the marginal benefit comprises the saved future production cost ρk as well as the durability subsidy *per unit* of the first-period stock, $-\tau_\phi/c_1^r$. Now, an increase in the number of firms reduces the individual stock c_1^r and thus raises the marginal benefit of durability by increasing the subsidy per unit. This effect provides the renting firms with the incentive to extend their product durability when the number of competitors increases ($\partial\phi^r/\partial n > 0$). Thus, if durability is subsidized a monopolist ($n = 1$) produces the smallest durability. By the same arguments, the monopolist provides the greatest durability if durability is taxed. Swan's independence result remains valid only for the special case in which the durability tax rate is zero or, equivalently, in which the tax payments of the renting firm don't depend on durability. This result is closely related to the analysis of Goering/Boyce (1999) who emphasis that durability depends on market structure if the tax payments of the renting firm aren't linear in the production rate. Both results identify the tax payments of producers as a further reason why Swan's independence result doesn't generally hold even in rental markets.[6]

4. Market Failure and First-Best Tax-Subsidy Schemes

Now turn to the analysis of efficient environmental regulation. To justify such regulation it is necessary to establish market failure under laissez-faire. For the present model this is done in proposition 2 where the values in the sales equilibrium and the social optimum are marked with s and o.

PROPOSITION 2. (*Market Failure under Laissez-Faire*)

Set all tax rates equal to zero.

(i) Then product durability under monopoly ($n = 1$) satisfies $\phi^s < \phi^r < \phi^o$ whereas durability under oligopoly ($n > 1$) satisfies $\phi^r < \phi^o$ as well as

$$\phi^s \gtreqless \phi^r \quad \Leftrightarrow \quad z \gtreqless 0 \qquad \textit{and} \qquad \phi^s \gtreqless \phi^o \quad \Leftrightarrow \quad z \gtreqless D_1'/\rho.$$

(ii) Then the stock of the durable good and the amount of solid waste in both the rental and the sales equilibrium may deviate from their efficient values in either direction.

The proof is presented in appendix A. For the rental case proposition 2 (i) generalizes the result under perfect competition derived by Runkel (1999a) to the case of imperfect competition: Since they ignore the external environmental costs of solid waste, the renting monopolist as well as the renting oligopolist simply minimize the production costs of durability and thus produce less durable goods than in the social optimum. Moreover, (i) provides two further insights. Firstly, as compared to the renting monopoly, in the selling monopoly product durability is further decreased below its efficient level since the selling monopolist has an incentive for planned obsolescence. Secondly, in a selling oligopoly durability may be greater than that chosen by renting oligopolists and greater than the socially optimal one since in this case the strategic effect is not zero and can even outweigh the obsolescence effect as well as the inefficiency due to the external cost. This result is somewhat surprising because in the political discussion of solid waste management it is often argued that requiring the firms to rent their products will always increase product durability (e.g. Soete (1997)). Proposition 2 (i) shows that this is true in monopoly but not in oligopoly. In oligopoly the commitment to rent the products may even reduce the durability since among other purposes the selling oligopolist uses durability to raise its own future market share and to reduce that of its rivals. Nevertheless, since there are several interacting distortion (externality, market power, obsolescence and strategic effect) the relationship between the efficient amount of waste and the industry amount of waste is ambiguous in both the rental and the sales equilibrium as indicated in proposition 2 (ii).

If the regulator aims at fully correcting for the market failure identified in proposition 2 then she has to compare the market equilibrium conditions with the efficiency conditions in the rows 1 to 3 of table I in order to determine those tax rates that make the market solution coincide with the socially optimal outcome. The resulting first-best tax-subsidy schemes are characterized in the rows 4 to 6 of table I. Since the number of tax rates exceeds the number of equations to be satisfied there are infinitely many first-best tax-subsidy scheme which is a somewhat unsatisfactory result. To reduce this diversity in a meaningful way, we exclude those first-best schemes which employ more than a minimum number of (non-zero) tax instruments. To be more specific, define an instrument as an element $\boldsymbol{\tau} \in \{(\tau_{w_1}, \tau_{w_2}), (\tau_{c_1}, \tau_{c_2}), (\tau_{y_1}, \tau_{y_2}), \tau_\phi\}$. Then it is easy to see that first-best schemes involve at least two instruments since any attempt to set all but one instrument equal to zero and solve 4 to 6 of table I for the remaining instrument implies a contradiction. All first-best tax-subsidy schemes consisting of two instruments are listed in table III which is obtained from the rows 4 to 6 of table I. Inspection of table III shows that the first-best optimum can be obtained by combining the waste taxes with any of the other instruments. If waste taxation is not available then each pair of the other instruments may be used to implement the social optimum.

These observations hold for the rental as well as for the sales equilibrium. Owing to the terms containing z, however, the tax rates differ in both cases. This arises the task of describing the properties of the first-best tax-subsidy

Table III. First-Best Tax-Subsidy Schemes Consisting of Two Instruments

with waste regulation	$\Big\{(\tau_{w_1},\tau_{w_2}),\,(\tau_{c_1},\tau_{c_2})\Big\}$	$\Big\{(\tau_{w_1},\tau_{w_2}),\,(\tau_{y_1},\tau_{y_2})\Big\}$	$\Big\{(\tau_{w_1},\tau_{w_2}),\,\tau_\phi\Big\}$
without waste regulation	$\Big\{(\tau_{y_1},\tau_{y_2}),\,(\tau_{c_1},\tau_{c_2})\Big\}$	$\Big\{(\tau_{y_1},\tau_{y_2}),\,\tau_\phi\Big\}$	$\Big\{(\tau_{c_1},\tau_{c_2}),\,\tau_\phi\Big\}$

schemes for the rental and the sales case. Since it will be quite formalistic to investigate all schemes listed in table III let us concentrate on the most important one.

PROPOSITION 3. (*First-Best Tax-Subsidy Scheme*)

Let $\tau_\phi = \tau_{y_1} = \tau_{y_2} = 0$. Then the first-best tax-subsidy scheme for the renting durable good industry is

$$\tau_{w_1} = D_1', \quad \tau_{c_1} = c_1^r P_1', \quad \tau_{w_2} + \tau_{c_2} = D_2' + c_2^r P_2' \tag{16}$$

whereas that for the selling durable good industry becomes

$$\tau_{w_1} = D_1' - \rho z, \quad \tau_{c_1} = c_1^s P_1' + \rho z, \quad \tau_{w_2} + \tau_{c_2} = D_2' + (c_2^s - \phi^s c_1^s) P_2'. \tag{17}$$

Proposition 3 is straightforward in view of the rows 4 to 6 of table I. The characterization of the first-best tax-subsidy scheme for the rental case in (16) confirms one's intuition: The environmental externality is completely internalized by a Pigouvian waste tax and the market imperfection is corrected by a subsidy on the stock of the durable. In contrast, the first-best tax-subsidy scheme for sales markets in (17) generally differs from the expected one owing to the obsolescence and the strategic role of product durability. As a consequence, with respect to waste taxation in sales markets overinternalization is just likely as underinternalization depending on the sign of z, i.e. $\tau_{w_1} \gtreqless D_1'$ if and only if $z \lesseqgtr 0$. The rationale of this result is as follows. If $z < 0$ (> 0) and if the solid waste in the selling industry is taxed according to the marginal damage, $\tau_{w_1} = D_1'$, then row 3 of table I shows that product durability is inefficiently small (great), hence the waste taxation is too lax (severe). To fully induce the efficient durability the waste tax needs to be raised beyond (lowered below) the marginal environmental damage. Note, that in case of monopoly z is negative and we obtain

COROLLARY 1. (*First-Best Waste Tax under Monopoly*)

If $\tau_\phi = \tau_{y_1} = \tau_{y_2} = 0$ then in a selling monopoly the first-best tax-subsidy scheme contains a waste tax $\tau_{w_1} > D_1'$.

Hence, to achieve a socially optimal outcome in a selling durable good monopoly the appropriate policy unambiguously requires to overinternalize the environmental damage in the first period.

To obtain further insights in the first-best regulation of the durable good industry, it is useful to compare it with that for a nondurable good industry investigated by Ebert (1992) or Kim/Chang (1993). After their approach is transformed to a two-period model the first-best tax-subsidy scheme for a nondurable good Cournot industry *without* an abatement technology reads[7]

$$\tilde{\tau}_{w_t} + \tilde{\tau}_{c_t} = D_t' + \tilde{c}_t P_t', \qquad t = 1, 2 \tag{18}$$

where the tilde marks the case of nondurable goods. The first-best tax-subsidy scheme for a nondurable good Cournot industry *with* an abatement technology is characterized by

$$\tilde{\tau}_{w_t} = D_t', \qquad \tilde{\tau}_{c_t} = \tilde{c}_t P_t', \qquad t = 1, 2. \tag{19}$$

If the firms rent their products then the first-best tax-subsidy scheme for the durable good industry in period 1 and in period 2 (equation (16)) equal the first best tax-subsidy scheme for a nondurable good industry with an abatement technology (equation (19) for $t = 1$) and without an abatement technology (equation (18) for $t = 2$), respectively. Of course, this result doesn't come as a surprise since in period 2 the durable good game is a degenerated nondurable good game without abatement whereas in period 1 the product durability has the same properties as an abatement technology: Increasing durability raises the production costs but also reduces the amount of solid waste and the associated environmental damage. Thus, if the firms rent their products then the first-best regulation of the durable good industry is analogous to that of the nondurable good industry. However, the first-best tax-subsidy scheme for the selling durable good industry differs from that of a nondurable good industry owing to the obsolescence and the strategic role of durability represented by z.

5. Second-Best Waste Taxation

The previous section showed that the regulator needs at least two instruments to obtain a first-best solution in the market equilibrium. In the real world, however, the regulator may have at her disposal environmental taxes only whereas stock, output and durability subsidies are not available due to political constraints. Then the market equilibrium necessarily deviates from the first-best optimum and second-best considerations are obvious: In such a scenario one might want to know how to set the rate of the waste tax in order to maximize social welfare in the durable good industry given that other instruments are not available.

Formally, for the purpose of deriving the second-best optimum the regulator makes use of the information that the stocks of the durable good and the product durability are functions $c_1(\tau_{w_1}, \tau_{w_2})$, $c_2(\tau_{w_1}, \tau_{w_2})$ and $\phi(\tau_{w_1}, \tau_{w_2})$ of the waste taxes owing to the equilibrium conditions in column 2, row 1 to 3 of table I. With this in mind the social welfare in (3) is maximized with respect to τ_{w_1} and τ_{w_2}. By using the equilibrium conditions the first-order conditions for the second-best optimum simplify to

$$\rho\frac{\partial c_2}{\partial \tau_{w_t}}\Big\{\tau_{w_2} - D_2' - (c_2 - \delta\phi c_1)P_2'\Big\} - c_1\frac{\partial \phi}{\partial \tau_{w_t}}\Big\{\tau_{w_1} - D_1' + \delta\rho z\Big\}$$
$$+ \frac{\partial c_1}{\partial \tau_{w_t}}\Big\{(1-\phi)\tau_{w_1} - (1-\phi)D_1' - c_1P_1' - \delta\rho\phi z\Big\} = 0, \quad t = 1, 2 \quad (20)$$

where the partial derivatives stand for the changes of the variables either in the rental or in the sales equilibrium. Solving (20) with respect to τ_{w_1} and τ_{w_2} and using the elasticity ε_x^t of the variable $x \,\epsilon\, \{c_1, c_2, \phi, w_1\}$ with respect to τ_{w_t} yields the second-best waste taxes listed in the row 7 and 8 of table I. From this information and the comparative dynamics of the equilibrium conditions in column 2, row 1 to 3 of table I we obtain

PROPOSITION 4. (*Second-Best Waste Taxation*)

(i) The second-best waste taxes for the renting durable good industry are

$$\tau_{w_1} = D_1' + c_1^r P_1' \frac{\varepsilon_{c_1}^1}{(1-\phi^r)\varepsilon_{w_1}^1} < D_1' \quad \textit{and} \quad \tau_{w_2} = D_2' + c_2^r P_2' < D_2'. \quad (21)$$

(ii) For the selling durable good industry there are parameter constellations for which the second-best waste taxes fall short of the marginal damage as well as parameter constellations for which the second-best waste taxes exceed the marginal damage and that even for $\varepsilon_{c_1}^1, \varepsilon_{w_1}^1 < 0$.

Before interpreting this proposition, a technical remark is in order: As the proof of proposition 4 in appendix B shows, the underinternalization result for the rental case relies on $\varepsilon_{c_1}^1$, $\varepsilon_{w_1}^1 < 0$ which in turn is implied by the assumption of decreasing returns to durability ($K'' > 0$). Furthermore, $K'' > 0$ is shown to be necessary to satisfy the second-order conditions for the profit maximum of a renting firm and thus it can't be disposed of. In contrast, $K'' < 0$ violates the second-order conditions and hence it is incorrect to conclude from (22) that for increasing returns to durability overinternalization may be second-best optimal in rental markets.

What's the intuition behind proposition 4? Underinternalization is optimal in rental markets since there are only the same two distortion at work as in a nondurable good industry, namely the environmental externality and the market imperfection. Thus, it may be argued as follows: If the regulator has at her disposal waste taxes only and if in both periods these taxes are set equal to the marginal environmental damage then in the rental equilibrium of the industry the environmental externality is completely internalized and product durability is efficient (see row 3 in table I) whereas the stock of the durable good and hence also the amount of solid waste in both periods are inefficiently small owing to the market imperfection $c_t P_t'$ (see row 1 and 2). If now the waste taxes in both periods are increased beyond the marginal damages then the social welfare will be decreased since durability increases beyond its efficient level (see $\partial\phi^r/\partial\tau_{w_1} > 0$ in appendix B) and the stock of the durable good further decreases below its efficient level in both periods (see $\partial c_t^r/\partial\tau_{w_t} < 0$ in appendix B). Hence, waste taxes greater than marginal damages can't be second-best optimal in rental markets. Following the usual second-best argument, however, society can gain from reducing the waste taxes below marginal damages since then product durability becomes inefficiently small, indeed, but at the same time the stock of the durable good becomes greater and hence more efficient.

Nevertheless, this reasoning doesn't apply to sales markets of durable goods since in such markets there is an additional distortion, namely the obsolescence and the strategic role of durability due to which durability generally deviates from its efficient level even if the environmental externality is completely internalized. Hence, overinternalization to be second-best optimal in sales markets may intuitively be explained by considering the special case of linear demand and damage functions ($P_t', D_t' =$ constant) in which the obsolescence effect dominates the strategic effect ($z < 0$). From row 3 in table I it then follows that even for waste taxes equal to the marginal damages not only the stock in both periods is inefficiently small but also is the product durability.[8] Now, an increase in the waste taxes beyond the marginal damages won't necessarily reduce the social welfare: the negative effect of further decreasing the stock of the durable good in both periods (see $\partial c_t^s/\partial\tau_{w_t} < 0$ in appendix B) is accompanied by the positive effect of increasing durability towards its efficient level (see $\partial\phi^s/\partial\tau_{w_1} > 0$ in appendix B). Depending on the relative size of these two effects the second-best waste taxes can even be greater than the marginal damages, which is in stark contrast to the rental case.

As mentioned in the introduction, the optimal taxation literature already provides some explanation why the second-best emission tax may exceed the marginal environmental damage. There are interesting links between this literature and the results of proposition 4: In showing overinternalization to be favourable for a nondurable good industry with a general abatement technolgy or an endogenous number of firms, Barnett (1980) eq. (12), Ebert (1992) eq. (8d) and Requate (1997) eq. (14) derive an expression for the second-best emission tax which for $\phi^r = 0$ exactly equals τ_{w_1} in (21). Hence, these authors need $\varepsilon_{c_1}^1 > 0$ or $\varepsilon_{w_1}^1 > 0$ to show the second-best emission tax to be greater than the marginal damage and they really find the theoretical possibility for that. As already argued, however, a positive correlation between the emission tax and the equilibrium production or the equilibrium amount of solid waste seems to be implausible.[9] The same criticism is true for the renting durable good model of Goering/Boyce (1999) and Runkel (1999b) in which overinternalization requires atypical reaction of the equilibrium durability or the equilibrium output. In the present model, however, proposition 4 (ii) provides an overinternalization result without imposing such debatable conditions.

6. Conclusion

Under the assumption of imperfect competition, this paper investigates a durable good industry in which the waste at the end of the product's life causes environmental damage. Several well known results from the industrial organization and the optimal taxation literature are challenged or extended: Firstly, the Swan independence result is shown to hold only if product durability is not directly regulated since otherwise the number of firms influences the marginal benefit of durability and therefore also durability itself. Secondly, if the regulator aims to reach the first-best social optimum in the selling industry through waste taxes and stock subsidies then this requires a waste tax which generally deviates from the marginal environmental damage and which is even greater than the marginal damage in case of a monopoly. Thirdly, if the regulator has at her disposal waste taxes only then the first-best optimum can't be reached and the second-best waste taxes fall short of the marginal damage if the firms rents their products; they may exceed the marginal damage, however, if the firms sell their products.

In contrast to the previous literature, the latter overinternalization result doesn't have counterintuitive implications. It should be emphasized, however, that it remains yet to be checked whether in reality the second-best waste taxes really exceed marginal damage. The present analysis merely shows that this is a possible outcome of theoretical analysis without standing in contrast to intuition. It doesn't exclude the case in which the second-best tax falls short of the marginal damage even if the firms sell their goods.

Notes

[1] Actually, Goering/Boyce (1999) state overinternalization to be second-best optimal if the demand and the decay functions are linear, the emissions depend only on output and the production technology exhibits increasing returns to durability. In Runkel (1999b) it is shown, however, that in this special case no industry equilibrium and consequently no second-best emission tax exist.

[2] Strictly speaking, this property of the marginal rental revenue is only required for $t = 2$. However, (1) for $t = 2$ implies (1) for $t = 1$.

[3] To see this, note that the rental profit in (5) can be transformed to $\Pi^r_{2i} = c_{2i}P(C_2) - \tilde{K}(c_{2i})$ with the linear cost function $\tilde{K}(c_{2i}) = (k+\tau_{w_2}+\tau_{c_2}+\tau_{y_2})c_{2i}+(k+\tau_{y_2})\phi_i c_{1i}$. Then the maximization of (5) is a special case of profit maximization in the nondurable good oligopoly (see e.g. Dixit (1986)) and according to a) and b) on p. 4 the Hahn conditions (1) imply uniqueness and stability for our second-period rental subgame, too.

[4] To apply Dixit's result, it is important to note that in period 2 the first-period decisions $\phi_j y_{1j}$ $(j = 1, \ldots, n)$ are given parameters. The maximization of Π^s_{2i} in 2 then represents a special case of profit maximization in Dixit's nondurable good oligopoly.

[5] For the purpose of comparison, the solution is derived by transforming the decision variables of firm i to c_{1i}, c_{2i} and ϕ_i. With this transformation Π^s_i equals (6) and in 1 firm i maximizes (6) with respect to c_{1i} and ϕ_i subject to the restrictions $c_{2j} = c_{2j}(\boldsymbol{\phi}\mathbf{c_1})$ $(j = 1, \ldots, n)$ which are determined by (12) and (13) or, alternatively, (14).

[6] See Schmalensee (1979), Muller/Peles (1990) and Goering (1993a,b) for further reasons why durablity of rented products may depend on the number of firms.

[7] (18) may be derived in the present model by ignoring the endogeneity of durability and therefore ignoring row 6 in table I and by setting $\phi = 0$ in the rows 4 and 5.

[8] ϕ^o and c^o_1 are determined by $P_1(nc^o_1) = G(\phi^o) + D'_1$ and $K'(\phi^o) = \rho k + D'_1$ with $G(x) := K(x) - xK'(x)$ and $G'(x) = -xK''(x) < 0$ whereas for $\tau_{w_1} = D'_1$ the equilibrium ϕ^s and c^s_1 satisfy $P_1(nc^s_1) = G(\phi^s) + D'_1 - c^s_1 P'_1$ and $K'(\phi^s) = \rho k + D'_1 + \rho z$. For $z < 0$ this implies $\phi^o > \phi^s$, $G(\phi^o) < G(\phi^s)$ and $c^o_1 > c^s_1$ owing to $P'_1 < 0$.

[9] It should be noted that in the case of an endogenous number of firms there might be an intuition for $\varepsilon^1_{c_1} > 0$ since an increase in the waste tax has the additional effect of reducing the number of firms and hence the firm's production may be increased (see Requate (1997), pp. 266). However, one of the conditions which ensure this outcome requires a non-convex demand function, and Requate (1997), p. 262 himself argues that '... both theoretically and empirically demand is more likely to be convex ... .'

Appendix

A. Proof of Proposition 2

For notational convenience define

$$F(x) := K(x) - \rho kx \quad \text{with } F'(x) = K'(x) - \rho k \quad \text{and } F''(x) = K''(x) > 0.$$

If all tax rates are zero then row 3 in table I shows that the efficient durability is determined by $F'(\phi^o) = D'_1$, the durability in the rental equilibrium satisfies $F'(\phi^r) = 0$ and the durability in the sales equilibrium is determined by $F'(\phi^s) = \rho z$. Under monopoly we obtain $\rho z < 0 < D'_1$ and thus $\phi^o > \phi^r > \phi^s$ owing to $F'' > 0$. Under oligopoly ρz is ambiguous in sign. $z \gtreqless 0$ is equivalent to $\phi^s \gtreqless \phi^r$ whereas $z \gtreqless D'_1/\rho$ is equivalent to $\phi^s \gtreqless \phi^o$ again owing to $F'' > 0$. This completes the proof of proposition 2 (i). To show (ii) note that due to the rows 1 and 2 of table I the efficiency conditions contains D'_t while the equilibrium conditions comprise $c_t P'_t$ and the terms containing z. D'_t, $c_t P'_t$ and z may exert opposite effects on the stock of the durable. Thus, the relationships between c^o_t, c^r_t and c^s_t as well as that between w^o_t, w^r_t and w^s_t are ambiguous $(t = 1, 2)$. (Q.E.D.)

B. Proof of the Propositions 4

In the proof of this proposition τ_ϕ, τ_{c_t} and τ_{y_t} are set equal to zero ($t = 1, 2$). As a preliminary we first show that presupposing $K'' > 0$ is necessary to satisfy the second-order conditions for the profit maximum of the renting firm. If firm i plays open-loop strategies it maximizes (6) with respect to c_{1i}, c_{2i} and ϕ_i taking as given the actions of its competitors (under closed-loop strategies firm i solves two problems but the second-order conditions will be the same). The Hessian determinant reads

$$\mathcal{H} = \begin{vmatrix} 2P_1' + c_{1i}P_1'' & 0 & 0 \\ 0 & \rho(2P_2' + c_{2i}P_2'') & 0 \\ 0 & 0 & -c_{1i}K'' \end{vmatrix}.$$

The second-order conditions require $\mathcal{H}$ to be negative definite. Hence, all diagonal elements of $\mathcal{H}$ have to be negative. For the first two rows this is satisfied by (1). However, the third row requires $K'' > 0$ as claimed.

With this preliminary, proposition 4 will now be proven. Firstly, consider the rental case. Totally differentiating the conditions for the rental equilibrium in column 2, row 1 to 3 of table I ($\delta = 0$) yields the comparative dynamic results

$$\frac{\partial c_1^r}{\partial \tau_{w_1}} = \frac{1-\phi^r}{(n+1)P_1' + nc_1^r P_1''} < 0, \qquad \frac{\partial c_2^r}{\partial \tau_{w_1}} = 0, \qquad \frac{\partial \phi^r}{\partial \tau_{w_1}} = \frac{1}{K''} > 0,$$
$$\frac{\partial c_1^r}{\partial \tau_{w_2}} = 0, \qquad \frac{\partial c_2^r}{\partial \tau_{w_2}} = \frac{1}{(n+1)P_2' + nc_2^r P_2''} < 0, \qquad \frac{\partial \phi^r}{\partial \tau_{w_2}} = 0,$$

where the signs of the derivatives are due to (1) and $K'' > 0$. These derivatives imply $\varepsilon_\phi^1 > 0$, $\varepsilon_{c_1}^1, \varepsilon_{c_2}^2 < 0$ and $\varepsilon_{c_2}^1 = \varepsilon_{c_1}^2 = \varepsilon_\phi^2 = \varepsilon_{w_1}^2 = 0$. Inserting the elasticities together with $\delta = 0$ into the equations for the second-best taxes in the rows 7 and 8 of table I yields (21). τ_{w_2} in (21) is smaller than D_2' owing to $c_2P_2' < 0$. By using the comparative dynamics, τ_{w_1} in (21) becomes

$$\tau_{w_1} = D_1' + c_1^r P_1' \frac{(1-\phi^r)K''}{(1-\phi^r)^2K'' - c_1^r[(n+1)P_1' + nc_1^r P_1'']}. \tag{22}$$

The second term in (22) is negative due to (1), $P_t' < 0$ and $K'' > 0$. Thus, in rental markets we obtain $\tau_{w_t} < D_t'$ ($t = 1, 2$) which completes the proof of proposition 4 (i).

To show proposition 4 (ii) for the sales case ($\delta = 1$), it is sufficient to provide an example in which the second-best waste taxes fall short of the marginal damage and an example in which at least one of the second-best waste taxes exceed the marginal damage. For this purpose, assume linear demand and damage functions: $P(C_t) = \alpha - \beta C_t$ and $D(W_t) = \gamma W_t$ with $\alpha, \beta, \gamma > 0$. Furthermore, consider quadratic unit costs $K(\phi) = \theta\phi^2$ with

$K'' = 2\theta > 0$. The conditions for the sales equilibrium in column 2, row 1 to 3 of table I then yield the comparative dynamic results

$$\frac{\partial c_1^s}{\partial \tau_{w_1}} = -\frac{1}{|J^s|}\Big\{2\theta\beta(1-\phi^s)(n+1) + \rho c_1^s \beta^2[(n+1)\Delta + \Gamma]\Big\} < 0, \quad (23)$$

$$\frac{\partial c_2^s}{\partial \tau_{w_1}} = \frac{1}{|J^s|}\Big\{c_1^s(n+1)\beta^2 - 2\theta\beta(1-\phi^s)\phi^s\Big\} \gtreqless 0, \quad (24)$$

$$\frac{\partial \phi^s}{\partial \tau_{w_1}} = \frac{1}{|J^s|}\Big\{(n+1)^2\beta^2 + \rho\phi^s\beta^2[(n+1)\Delta + \Gamma]\Big\} > 0, \quad (25)$$

$$\frac{\partial c_1^s}{\partial \tau_{w_2}} = \frac{2}{|J^s|}\,\rho\phi^s\theta\beta\Gamma < 0, \qquad \frac{\partial \phi^s}{\partial \tau_{w_2}} = \frac{1}{|J^s|}\,\rho(n+1)\beta^2\Gamma < 0, \quad (26)$$

$$\frac{\partial c_2^s}{\partial \tau_{w_2}} = -\frac{1}{|J^s|}\Big\{2\theta\beta(n+1) + \rho c_1^s(n+1)\beta^2\Delta + 2\theta\beta\rho\phi^{s2}\Delta\Big\} < 0, \quad (27)$$

with $\Delta := n/(n+1) > 0$, $\Gamma := (1-n)/(n+1) < 0$ and the Jacobian determinant $|J^s| := 2\theta(n+1)^2\beta^2 + \rho c_1^s(n+1)\beta^3[(n+1)\Delta+\Gamma] + 2\theta\beta^2\rho\phi^{s2}[(n+1)\Delta + \Gamma] > 0$. $\varepsilon^1_{c_1}, \varepsilon^1_{w_1} < 0$ follows from (23) and (25). For the special case $n = 1$ (monopoly) we obtain $\Gamma = 0$ and thus $\partial c_1^s/\partial\tau_{w_2} = \partial\phi^s/\partial\tau_{w_2} = 0$ as well as $\varepsilon^2_{c_1} = \varepsilon^2_{\phi} = \varepsilon^2_{w_1} = 0$. Inserting this together with $z = -\phi^s c_1^s\beta/2$ and $\delta = 1$ into the rows 7 and 8 of table I and rearranging terms yields

$$\tau_{w_1} = \gamma - \beta c_1^s \frac{2\theta\beta(1-\phi^s)(2+\rho\phi^{s2}) - \rho c_1\beta^2(2\phi^s - 1)}{4\theta\beta(1-\phi^s)^2 + 4c_1^s\beta^2 + \rho c_1^s\beta^2}, \quad (28)$$

$$\tau_{w_2} = \gamma - \beta(c_2^s - \phi^s c_1^s). \quad (29)$$

The second-best waste tax in period 2 falls short of the marginal damage γ. However, the relationship between the second-best waste tax and the marginal damage in period 1 is ambiguous owing to $\phi^s \gtreqless 1/2$. To illustrate this statement, solve the model for two parameter constellations:[1] For $n = 1$, $\rho = 0.99$, $\alpha = 10$, $\beta = \theta = k = 1$ and $\gamma = 1.5$ we obtain $c_1^s \approx 4.621$, $c_2^s \approx 7.499$, $\phi^s \approx 0.456$ and $\tau_{w_1} \approx 0.967 < 1.5$, $\tau_{w_2} \approx -3.891 < 1.5$. Hence for this constellation underinternalization is second-best optimal. However, if the marginal damage is c.p. increased to $\gamma = 2$ then $c_1^s \approx 4.171$, $c_2^s \approx 7.000$, $\phi^s \approx 0.815$ and $\tau_{w_1} \approx 2.321 > 2$, $\tau_{w_2} \approx -1.602 < 2$. Thus, overinternalization is now second-best optimal in period 1. (Q.E.D.)

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[1] Details on these calculations can be obtained upon request.

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Supplementary Material (not to be published)

1. Comparative Dynamics for the Social Optimum (Table II)

Define $P_{t,\kappa} := \partial P_t/\partial\kappa_P$ and $D'_{t,\kappa} := \partial D'_t/\partial\kappa_D$. Totally differentiating the conditions in column 1, row 1 to 3 of table I yields the matrix equation

$$\begin{pmatrix} nP_1' - (1-\phi)^2 nD_1'' & 0 & (1-\phi)nc_1D_1'' \\ 0 & nP_2' - nD_2'' & 0 \\ -(1-\phi)nD_1'' & 0 & K'' + nc_1D_1'' \end{pmatrix} \begin{pmatrix} dc_1 \\ dc_2 \\ d\phi \end{pmatrix}$$

$$= \begin{pmatrix} -P_{1,\kappa} & (1-\phi)D'_{1,\kappa} & -\phi k & -[c_1P_1' - (1-\phi)^2c_1D_1''] \\ -P_{2,\kappa} & D'_{2,\kappa} & 0 & -[c_2P_2' - c_2D_2''] \\ 0 & D'_{1,\kappa} & k & (1-\phi)c_1D_1'' \end{pmatrix} \begin{pmatrix} d\kappa_P \\ d\kappa_D \\ d\rho \\ d\phi \end{pmatrix}$$

with the Jacobian determinant $|J^o| = n^2K''[P_2' - D_2''][P_1' - (1-\phi)^2D_1''] + n^3c_1D_1''P_1'[P_2' - D_2''] > 0$ owing to $K'' > 0$. The results in table II are easily obtained by applying Cramer's rule to the above matrix equation.

2. Proving the Equations (11) to (14)

In the notation of Dixit (1986), for all $i = 1, \dots, n$ define

$$a_i := 2P'\Big(\sum_{j=1}^n (y_{2j} + \phi_j y_{1j})\Big) + y_{2i}P''\Big(\sum_{j=1}^n (y_{2j} + \phi_j y_{1j})\Big),$$

$$b_i := P'\Big(\sum_{j=1}^n (y_{2j} + \phi_j y_{1j})\Big) + y_{2i}P''\Big(\sum_{j=1}^n (y_{2j} + \phi_j y_{1j})\Big),$$

$$\Gamma := 1 + \sum_{j=1}^n \frac{b_j}{a_j - b_j},$$

and

$$\mu^i(y_{21}, \dots, y_{2n}; \theta_i) := P\Big(\sum_{j=1}^n (y_{2j} + \phi_j y_{1j})\Big) - (k + \tau_{w_2} + \tau_{c_2} + \tau_{y_2}) + y_{2i}P'\Big(\sum_{j=1}^n (y_{2j} + \phi_j y_{1j})\Big) \qquad (30)$$

with the exogenous parameters $\theta_i \,\epsilon\, \{\phi_1 y_{11}, \ldots, \phi_n y_{1n}\}$. Thus, (10) becomes $\mu^i(y_{21}, \ldots, y_{2n}; \theta_i) = 0$ for $i = 1, \ldots, n$. With this notations Dixit (1986), eq. (41) shows that

$$dy_{2i} = -\frac{\mu_\theta^i d\theta_i}{a_i - b_i} + \frac{b_i}{\Gamma(a_i - b_i)} \sum_{j=1}^{n} \frac{\mu_\theta^j d\theta_j}{a_j - b_j}, \qquad i = 1, \ldots, n \tag{31}$$

where $\mu_\theta^i := \partial\mu^i/\partial\theta_i$ all $i = 1, \ldots, n$. For a particular $\ell \,\epsilon\, \{1, \ldots, n\}$ the partial derivative $\partial y_{2i}/\partial\phi_\ell y_{1\ell}$ is obtained by setting $\theta_i = \phi_\ell y_{1\ell}$ all $i = 1, \ldots, n$. From (30) we then obtain

$$\mu_\theta^i = \frac{\partial\mu^i}{\partial\phi_\ell y_{1\ell}} = P_2' + y_{2i}P_2'', \qquad i = 1, \ldots, n.$$

Inserting this together with the above definitions in (31) after some rearrangements yields

$$\frac{\partial y_{2i}}{\partial\phi_\ell y_{1\ell}} = -\frac{P_2' + y_{2i}P_2''}{(n+1)P_2' + Y_2 P_2''}, \qquad i = 1, \ldots, n$$

which is true for all $\ell \,\epsilon\, \{1, \ldots, n\}$. Finally, changing the indices proves (11) in the text. (12) to (14) are easily obtained by using the definitions of c_{ti} and C_2^{-i}.

3. Profit Maximization of the Selling Firm

If firm i sells its output then in period 1 it faces the problem of

$$\begin{aligned} \max_{y_{1i},\phi_i} \Pi_i^s \; &= \; y_{1i} \left\{ P\Big(\sum_{j=1}^{n} y_{1j}\Big) - K(\phi_i) - (1-\phi_i)\tau_{w_1} - \tau_{c_1} - \tau_{y_1} \right\} - \phi_i \tau_\phi \\ &+ \; \rho \left\{ \Big(y_{2i}(\boldsymbol{\phi y_1}) + \phi_i y_{1i}\Big) \left\{ P\Big(\sum_{j=1}^{n} (y_{2j}(\boldsymbol{\phi y_1}) + \phi_j y_{1j})\Big) - \tau_{w_2} \right.\right. \\ &\left.\left. - \; \tau_{c_2} \right\} - (k + \tau_{y_2}) y_{2i}(\boldsymbol{\phi y_1}) \right\} \end{aligned} \tag{32}$$

where $y_{2j}(\boldsymbol{\phi y_1})$ $(j = 1, \ldots, n)$ is determined by (10) with the partial derivatives captured by (11). For comparsion purpose it is useful to transform the decision variables of firm i from y_{1i}, ϕ_i to $c_{1i} := y_{1i}$, ϕ_i and to define $c_{2i}(\boldsymbol{\phi c_1}) := y_{2i}(\boldsymbol{\phi c_1}) + \phi_i c_{1i} = y_{2i}(\boldsymbol{\phi y_1}) + \phi_i y_{1i}$ $(i = 1, \ldots, n)$. The problem

(32) then becomes

$$\max_{c_{1i},\phi_i} \Pi_i^s = c_{1i}\left\{P\Big(\sum_{j=1}^{n} c_{1j}\Big) - K(\phi_i) - (1-\phi_i)\tau_{w_1} - \tau_{c_1} - \tau_{y_1}\right\} - \phi_i\tau_\phi$$
$$+ \rho\left\{c_{2i}(\boldsymbol{\phi c_1})\left\{P\Big(\sum_{j=1}^{n} c_{2j}(\boldsymbol{\phi c_1})\Big) - \tau_{w_2} - \tau_{c_2}\right\}\right.$$
$$\left. - (k+\tau_{y_2})\Big(c_{2i}(\boldsymbol{\phi c_1}) - \phi_i c_{1i}\Big)\right\} \qquad (33)$$

where $c_{2j}(\boldsymbol{\phi c_1})$ $(j = 1, \ldots, n)$ is determined by (10) with the partial derivatives captured by (12) to (14). Differentiating Π_i^s in (33) with respect to c_{1i} and ϕ_i, using the equilibrium conditions (10) to cancel common terms and setting the resulting expressions equal to zero yields the first-order conditions

$$P(C_1) + c_{1i}P'(C_1) + \rho k\phi_i = K(\phi_i) + (1-\phi_i)\tau_{w_1} + \tau_{c_1} + \tau_{y_1} - \rho\phi_i\tau_{y_2} - \rho\phi_i z \qquad (34)$$

and

$$K'(\phi_i) = \rho k + \tau_{w_1} + \rho\tau_{y_2} - \tau_\phi/c_{1i} + \rho z \qquad (35)$$

with

$$z := \phi_i c_{1i}P'(C_2)\frac{\partial c_{2i}}{\partial\phi_i c_{1i}} + c_{2i}P'(C_2)\frac{\partial C_2^{-i}}{\partial\phi_i c_{1i}}$$

where $\partial c_{2i}/\partial\phi_i c_{1i}$ and $\partial C_2^{-i}/\partial\phi_i c_{1i}$ are determined by (12) and (14), respectively. Now, the condition for a symmetric sales equilibrium are obtained by using $\phi_i = \phi$, $c_{ti} = c_t$ and $C_t = nc_t$ $(t = 1, 2)$ in the equation (10), (34) and (35). The resulting expressions coincides with the column 2, row 1 to 3 in table I. Note, that the index i in $\partial c_{2i}/\partial\phi_i c_{1i}$ and $\partial C_2^{-i}/\partial\phi_i c_{1i}$ can be suppressed owing to the symmetry of the equilibrium.