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Andreas Wagener

University of Siegen

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Andreas WAGENER

Fachbereich 5, VWL IV
University of Siegen
Hoelderlinstrasse 3
D-57068 Siegen, Germany

Phone: +49 - 271 - 740 3164
Fax: +49 - 271 - 740 2732
e-mail: wagener@wap-server.fb5.uni-siegen.de

Abstract: We model a pay-as-you-go (PAYG) pension system as a series of incomplete intergenerational contracts. Each generation pays a pension to its parents as the price for a premortal transferral of economic property rights. The terms of this intergenerational trade are fixed in a social contract, which due to its long-term nature is incomplete and likely to be renegotiated after some of the initial uncertainty has been resolved. In between, however, investments and education efforts have to be carried out which affect the value of the economic resources to be transferred between generations. This set-up creates a number of intergenerational externalities (including a canonical hold-up problem) which may contribute to the explanation of those problems that real-world PAYG public pension systems currently face.

Keywords: Pay-as-you-go Pension Systems, Incomplete Contracts, Social Contracts
JEL-Classification: H55, L14, D71

1. Introduction

The central idea of this paper is to model pay-as-you-go (PAYG) pension systems as a series of incomplete intergenerational social contracts. The parties of these contracts are members of different generations who - very generally - agree upon the terms of some intergenerational trade between them. Just like ordinary contracts in commodity trade the intergenerational contracts specify prices and (quantities of) trade objects. The trade objects of intergenerational trade are ownership rights to the economy's resources. Each generation creates and owns parts of the economy's means of production (say, the capital stock). When getting old, it can sell its ownership rights, which include both control and residual rights, to its children's generation and retire instead of working until death and then bequeathing its wealth to its descendants. The price to be paid by the children for the "premortal" transfer of ownership rights is the old-age support for their parents. This support has to be financed out of current income which constitutes the PAYG property of old-age support in our approach. Intergenerational trade may be advantageous for both generations and socially efficient because, e.g., the productivity of the young is higher than that of the elderly (due to diminishing physical strength or mental power during old age).

Within a society, generations are locked-in to each other: they each only have one "trading partner" and their investments are relation specific. As a rule, outside options are not available to any of them and hence their living together can only be governed by social contracting and not, e.g., by competition. Social contracting which is intended to rule very long-term relationships (periods of some 30 years, say) faces three generic difficulties. The first is that the many contingencies which arise as time goes by cannot be fully dealt with when the contract is initially drawn up. As a consequence social contracts are highly incomplete in the sense that they cannot be written in contingent terms. Furthermore, renegotiation of the contract cannot be credibly ruled out *ex ante*. Second, the value of the means of production in an economy, which may be different seen from the viewpoints of two generations even if evaluated at the same point of time, depends on a number of variables such as physical components, technologies, skills, organizational features and ownership rights. These are to a great extent the result of the generations' investments into human and non-human capital. In most cases, these investments are sunk once carried out. Furthermore, their complexity renders them non-contractible. If renegotiation is possible, this may create intergenerational hold-up problems. Third, a contract between two generations does not only affect their relation, but due to the overlapping of generations may have repercussions on third generations as well (e.g., through the capital stock left to them) and thus influence the long-run economic prospects of a society.

The fact that social contracts are incomplete, long-term and prone to renegotiation affects the behaviour of the contracting generations, i.e. their specific investments. Hold-up problems and underinvestment are likely to occur. This automatically raises the question whether cleverly designed social contracts (i.e., PAYG systems) exist which overcome these troubles.

In this paper we set up an OLG model of a closed economy which incorporates these features. As a first attempt we stick rather closely to the I/O literature which provides considerable insights for the design of contracts in long-term settings with *ex-ante* non-contractibilities.

Our aims are

- to show how PAYG pension schemes can be understood as bilateral intergenerational contracts;
- to highlight the specific properties of such contracts: incompleteness and the possibility of renegotiation;
- to identify intergenerational externalities which may distort investments away from their efficient levels;
- to assess the chances of creating an optimal PAYG system.

The rest of the paper is organized in six parts: In Section 2 we give a broad motivation for our approach. Section 3 puts our ideas into the formal framework of an OLG model. Section 4 discusses the renegotiation game. In Section 5 we derive conditions for the optimal behaviour of the generations in our model and for efficient behaviour. Comparing both in Section 6, we identify a number of intergenerational externalities (including an intergenerational hold-up problem) which may distort economic decisions away from their efficient levels. Some implications and directions for future research are discussed in the concluding Section 7.

2. Further Motivation

Almost all public pension plans in industrialized countries are financed on a PAYG base, and in almost all countries these PAYG systems are wobbling or expected to become so in near future. Several remedies have been discussed and several countries have gone through minor or major modifications of their PAYG system. To keep actuarial balance of a PAYG system in an aging society requires rising contribution rates, lower pension payments or both (see Weizsäcker (1995) for a discussion). Diamond (1996) discusses the indexation of the retirement age, investments of trust funds in private securities, partial privatization of social security, and mandating individual or employer-provided retirement savings. All reforms have in common that at least one generation incurs losses as compared to the current system (the same would happen with reversed signs if economic conditions changed in a favourable direction). Generally, one may wish to divide such losses (or benefits) amongst the generations.

In modern societies, this works through some bargaining between the generations currently alive and involved in the PAYG system.[1] We will interpret this kind of bargaining as the renegotiation of a social contract which has been agreed upon by the generations at some earlier point in time.

It is well-known that PAYG public pension systems are not neutral with respect to individual behaviour. Their existence influences households' savings, labour supply, investments, consumption streams and reproductive behaviour. Reforms in the PAYG system often hit individuals after a considerable part of their lifetime economic decisions has already been irreversibly made; naturally, the older an individual, the greater is this part. Previous economic decisions may have been based on the assumption that the pension system will remain at work unchangedly and forever. However, with rational agents, this is not very plausible. Agents can foresee that economic or demographic conditions will change (although they naturally cannot foresee the exact state of the future world) and that the actuarial assumptions the current public pension system is based upon may not hold in the future. Agents anticipate that the pension system may be subject to change or, with other words, that the social contract may be renegotiated. This prospect will affect their behaviour from the outset, i.e., decisions are made under the assumption of an uncertain pension system.

In this paper we set up a model which elaborates on this idea. Before presenting the model, we give some further motivation for our approach and put it into perspective with the literature.

1. The public finance literature offers several positive and normative explanations for PAYG public pension systems (for a survey see Verbon (1993)):

- In Browning (1975) a PAYG system is the result of a majority voting rule with the old out-voting the young. In this setting, the median-voter PAYG system is socially inefficient. The Browning approach is not very satisfactory for a number of reasons. As Veall (1986) points out, social security is the only form of redistributing wealth in the Browning model. If there were other instruments (such as taxes), no voting equilibrium would exist. Verbon (1988) shows that in a representative democracy PAYG systems may not emerge even if the elderly are in the majority. Furthermore, the one-to-one mapping that the young oppose PAYG systems whereas the elderly favour them does not hold empirically. This observation indicates that in order to be in harmony with the polls a positive explanation of PAYG systems it cannot be based on one-sided supports only.
- Diamond/Mirrlees (1978) argue that moral hazard and adverse selection problems in retirement income insurance can (only) be alleviated by compulsory participation. While this highlights some risk sharing issues in the pension problems, this does, however, not explain PAYG systems, but only government intervention in the provision of pensions.

[1] Sometimes this bargaining is even seen as a kind of *generational combat* (The Economist, January 11, 1007, p. 49).

- Diamond (1977) argues that individiuals neglect private savings for their old age due to uninformedness and irrationality. Therefore, the government should set up a forced-savings programme. Again, there is no explanation for a PAYG system, but only for government intervention. Furthermore, the argument involves paternalism (see Veall (1986) for this point) and irrationality.
- Veall (1986) presents an intergenerational game with altruistic generations where strategic considerations make individuals stop saving and rely on future generations' altruism. He then shows that with a compulsory PAYG pension plan an efficient intertemporal allocation can be implemented. The problem with Veall's approach is that it does not purely rely on the Nash equilibrium concept but, such as Samuelson (1958), on reciprocity in the sense that each generation assumes that a \$1 reduction of transfers to its elders will once be paid back by its children in form of an equal \$1 reduction of their transfer. In other words, the current generations behave as if they were ascribed the right to establish a binding rule for future generations. How such a rule can be enforced is a question left open.
- A number of papers argue in favour of a PAYG system for the reason of intergenerational efficiency. Breyer (1989) shows that it is not possible to find a Pareto improving transformation of a PAYG pension system into a fully funded system if contributions to the PAYG system are lump sum. There is no way to raise the necessary funds to finance the transition period without making at least one generation worse off as compared to a continuation of the PAYG system. This result has been qualified by several authors (see e.g. Homburg (1990) or Fenge/Schwager (1996)) leaving the impression that (to say the least) no clear-cut case can be made for the superiority of PAYG systems. This especially holds if contributions to the public pensions systems are distortionary.

To summarize, there seems to be no fully convincing theory which can explain the widespread and stable existence of PAYG pension systems without relying on altruism or behavioural assumptions different from economic rationality.

2. As Verbon (1988) points out, the public pension systems of industrialized countries are due to permanent changes and discussions. Hardly any government has left the pension system as it found it.[2] This empirical observation is in contrast with the theoretical literature (e.g., with Browning (1975), Boadway/Wildasin (1989) and Veall (1986)) where it is implicitly assumed that the (PAYG) system once agreed upon will continue unaltered in the future. This assumption is not tenable. A realistic approach to the public pension problem should incorporate an explanation for changes and alterations in the system. Such an explanation could be that at the time when the decision on the pension scheme was made certain contingencies could not be or were not foreseen. Sticking to the old agreement then would certainly not be optimal.

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2 E.g., the contribution rate (as a percentage of gross income) to the German old-age insurance system was changed 13 times between 1980 and 1997.

3. The German PAYG system of old age insurance is called *Generationenvertrag* (intergenerational contract). This "contract" has some strange features. First, it is not a contract in the sense of the Civil Law. The German Supreme Court put this very clearly when stating that payments to the PAYG system during working life do not constitute any claim for a certain amount of old age pensions during retirement. Moreover, anybody who enters the PAYG system "cannot expect the legal prescriptions concerning the payments of the pension insurance to remain unchangedly valid for ever. (...) Thus, changes in economic conditions or as well in the ratio between pensioneers and those generations who, while still working, bear the contributions to the pension insurance, will give scope or necessity for various realignments." (BVerfGE 58, 123, own translation). Note that this serves as a justification for the permanent changes and alterations which occur in the German public pensions system. Second, however, the PAYG system is a contract in the sense that "those who are working take care for the retirement income of the then old, and thereby earn the claim to be supported in their own old age by those who are then working." (ibid.) Note that there is no link between the premiums paid during working age and the pension payments received when old.[3] The only guarantee the (German) PAYG system offers is that there will be a *chance* for each generation to reach an agreement with its children's generation concerning the support during old age (see also Brunner/Wickström (1993)). This guarantee is fairly weak, to say the least.
Without any further commitment, PAYG systems cannot be seen as constitutional social contracts ensuring an infinite chain of transfers from young to old. As has been recognized by Samuelson (1958) already, it is in general not possible to bind future generations in an OLG model. The reason is that the young receive nothing in exchange for their support to the old - except the vague hope that somebody will perhaps take care of them when they are old.[4] Each generation therefore has an incentive to repudiate the payments to the current old (as Wallace (1980) points out, this is an equilibrium of the respective transfer game).

How can we interpret PAYG systems then? One possibility is to see a PAYG system as a series of unilateral and voluntary transfers from the young to the old. This interpretation purely relies on altruism which - from an economic perspective - is not a convincing rationale. A second strand of the literature adds to the social contract some ingredients which ensure enforcement of the contract by all subsequent generations:

- In Hansson/Stuart (1989) generations are altruistic both towards earlier and future generations. The initial generations in the Hansson/Stuart OLG model agree on a Ponzi-

[3] This is true at least in a formal sense. *De facto*, pension payments in the German PAYG system do depend (among others) in a weakly monotonic way on amount and duration of premium payments. The "pension formula", however, is not fixed and may be subject to sudden changes. Furthermore, there is no obligation for the state to spend contributions to the public pension system for pension payments entirely. This opens floodgates for any kind of "abuse" of old-age provisions.

[4] A nice illustration of the vagueness of this hope may be that in the United States more people under 35 believe in flying saucers than that they will ever get a Social Security cheque when they are old (The Economist, January 11, 1997, p. 49).

type PAYG pension system which, as altruism declines with time distance, implies non-decreasing transfers and decreasing savings over time. Once set up, this suboptimal PAYG system will run forever as at any future point of time the old generation will refuse to change the social contract. As, by assumption, the social contract is constitutional and can only be altered with unanimous consent, the PAYG system will survive. The unanimity rule serves as a commitment device to the PAYG system. In many countries the public pension scheme is not constitutional, but can be changed with simple majority. in a growing economy, the young are in the majority and thus would vote for abolition of the scheme. Allowing for different or changing bargaining powers would destroy the Hansson/Stuart result (see Verbon (1993) for this point).

- In Esteban/Sakovics (1993) the intergenerational trust problem of how to bind future generations to transfers once agreed upon in a social contract is solved by means of institutions which are costly to set up and to change. These costs suffice to sustain positive intergenerational PAYG transfers both as cooperative and non-cooperative equilibria (although in general at suboptimally low levels).

The approach followed in this paper is quite different. PAYG pension systems are assumed neither to emerge from altruism nor to survive via institutional inertia. Our interpretation is that PAYG systems are based on a series of at-will agreements between generations. In the literature, there exist two contractual approaches to PAYG systems which are similar to ours:

- The first is Kotlikoff et al. (1988). In their model payments from young to old generations are the price paid by the young to make the old stick to some social contract they would otherwise abrogate. The social contract (which in the Kotlikoff et al. model prescribes low capital taxation) itself is a traded asset. Selling it from generation to generation ensures its permanent enforcement. Thus, time-inconsistency problems are overcome.
- The second approach is Cigno (1993). In this paper a family is interpreted as a credit system. Individuals only earn income in the middle, working period of their lives. Before and after (i.e., during youth and old age) they have to rely on transfers from their parents and children, respectively. In such a framework the middle-aged may lend to the young who pay back this loan one period later when the young borrowers are middle-aged and the initially middle-aged lenders have become old. Cigno (1993) shows that such an intra-family deal combined with the threat upheld by each generation never to pay anything to somebody who did not comply with the rules of the game is an efficient and subgame-perfect (non-cooperative) equilibrium of the family game. Pensions in this model may be seen as the return-on-investment in children.[5]

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[5] Furthermore, Cigno (1983) shows that this efficient equilibrium is destroyed if (i) there is an accessible capital market (even if it offers lower rates of return than the family system does) or (ii) if there is a compulsory public pension system. Both theoretical findings fit well with the empirical observation of the break-up of family ties in modern societies.

Both papers cited do not rely on Barro-type altruistic preferences but assume purely self-interested individuals. So does our paper which differs from those approaches in other respects: Here, pension payments are the price to be paid by the young in exchange for property rights owned by the old. The intergenerational trade-object thus is economic resources and not, as in Kotlikoff et al. (1988), a social contract itself. In Cigno (1993), individuals get entitled to pension payments during old-age if (and only if) they once raised and supported children. Our approach is less dynastic and more profane. Pensions are not the reward for some (socially beneficial) deeds in the past but merely the ordinary price in a one-to-one deal.[6]

The idea of bilateral intergenerational contracts circumvents both the problem of binding future generations and the reliance on altruism. Instead, we get a purely economic interpretation of a pension system.

4. In our interpretation pension payments are the price for the transferral of the economy's ownership rights from an old generation to a younger one. This fancy idea is perhaps not as far-fetched as it may seem at first sight:

a) During their working time individuals invest in human and non-human capital and accumulate wealth or, more generally, ownership rights to economic resources. Personal wealth is increasing with age. The old generation owns considerable part of an economy's capital stock. "Selling" this capital stock to the younger generations may serve as a source of income during old age. For illustrative examples consider entrepreneurs, farmers or house owners. Intergenerational contract between older and younger generation then specify the conditions under which the sons and daughters take over the firm, the farm or the house, including especially the income of the retired firm owner, the duties to support the parents during old age or the right for a lifetime residence in the house. Such types of contracts may be socially efficient if the use of the asset by the young generation is more profitable than the use by the elderly (e.g., due to diminishing physical strength or mental power during old age).

b) Interpret the trade object of the social contract as the age of retirement of the elderly. Suppose that the old age period (say, the age between 60 and 90) consists of a first section where the old aged still work and a retirement part. If labour demand is below labour supply, the higher is the age of retirement of the older generation, the higher is their lifetime income from labour supply and, as less jobs are available for the younger generations, the lower is labour income for the children's generation. Therefore both generations may agree on a social contract which specifies a low age of retirement with

6 Note that this has important consequences on the institutional framework needed to enforce a PAYG pension system. In Cigno (1993) the intra-family credit system is self-enforcing, i.e. there is no need for further institutions. In our approach we (at least) need some agency which guarantees that generations really stick to the terms of their contracts.

rather high pension payments.[7] To pay a pension in exchange for a job is a common practice in all OECD countries (for a survey and discussion, see Casey (1989)). In 1995, the German PAYG pension system paid 18.6 billion DM as early retirement pensions, caching unemployment.

For (intergenerational) trade to be welfare improving, the valuation of the traded goods has to be different for the seller (i.e. the old generation) and the buyer (the young generation). This is quite clear in the above case b), but it also may hold in case a): When employed by the younger generation, the same amount of physical capital stock (a firm, say) may yield different returns within the next 30 years as compared to the case where it is run by the old generation.

5. As Rogerson (1992) points out, long-term contractual relationships (between whatever parties) often face two problems: First, the exact form of the transaction to be carried out in some distant future cannot be specified with certainty *ex ante* because it depends on uncertain parameters which cannot be completely described and thus cannot be contracted upon. Second, to prepare for the transaction the contract parties must make specific investments which are sufficiently complex to be contractible. These two problems lead to contractual incompleteness (see Hart (1995) for an excellent introduction) and have opposite implications for contract design:

- The latter problem calls for rigid contracts as otherwise the contracting parties must fear that their investment efforts may be exploited by the opposite party in the sense that in bargaining (renegotiation) subsequent to investments the terms of trade cannot be prevented from being changed in favour of the "lazy" party. I.e., without a precise contract, underinvestment is very likely to occur (this is the classical hold-up problem).
- The former of the two problems mentioned above, however, calls for relatively loose contracts in order not to sacrifice the possibility to agree on new terms of trade which are beneficial to both parties in case some "favourable" state of Nature occurs.

Economic contract theory has devoted considerable effort how to deal with these conflicting requirements. In a seminal paper, Hart/Moore (1988) argue that contractual incompleteness together with the impossibility to credibly rule out renegotiations of long-term contracts *ex ante* lead to underinvestment. Although several papers argue that, if more complex contracts than assumed in Hart/Moore (1988) can be written, the hold-up problem can be overcome (for a very general treatment see Rogerson (1992)), the general message of Hart/Moore (1988) seems to be undisputed: If contract complexity is limited, then, as a rule, non-optimal investment are very likely.

Whereas these result have been derived in the industrial organization literature, we think that similar problems also arise in the field of social contracting. Social contracts typically deal with long-time relationships between the members of a society. The most prominent area of social

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[7] If jobs are understood as a special type of ownership right to an economy's resources, this interpretation can be seen in a line with case a).

contracting with a long philosophical tradition is constitutional choice (for a survey see Boucher/Kelly (1994)). The seminal works of Rawls, Harsanyi and Buchanan/Tullock brought this topic to the attention of economists who look for justifications for particular rules to be applied in social choice problems. The central instrument in constitutional social contracting is the *veil of ignorance*: By undertaking such a *gedankenexperiment* the members of the constitutional assembly artificially create a situation of equality which leads to unanimous decisions on the social choice rules to be applied in front of the veil (see Eichberger/Pethig (1994)). The need for writing a constitution as a collection of rules has some similarity with the incomplete contract approach. First, (social) contracts will only be valid if all parties unanimously agree upon them. Secondly and more important, in both situations it is not possible to write a (social) contract which deals with all contingencies. The idea, that constitutions can be in fact be seen as incomplete social contracts, can (albeit in rather vague form) be found in Tirole (1994, p.16) and Caillaud/Jullien/Picard (1996, p.690). However, there are two decisive differences in both approaches: First, constitutions (at least those agreed upon behind the veil of ignorance) are not expected to be revised. The social contracts we consider are prone to renegotiation and social contractors will anticipate this in the contractual phase. Second, Rawlsian constitutional choice faces the difficulty of how to deal with people acting not in accordance with the principles agreed upon behind the veil of ignorance. This problem (which is related to the time inconsistency problem) is not an issue in the social contracts discussed here; they are self-enforcing in the sense that rational expected utility maximizers find it advantageous to stick to the contract terms.

3. The model: General framework, time structure and social contracts

We consider an OLG model where every generation lives for three periods (youth, working period and retirement). We label ages by superscripts 1,2 and 3 respectively. Each generation is represented by a single individual. The generation which is young in period t will be called generation t. By C_t^1, C_t^2 and C_t^3 we denote consumption of generation t in its youth, working period and retirement period, respectively. Lifetime utility of generation t is given by an intertemporal von-Neumann-Morgenstern utility function $u^t = U(C_t^1, C_t^2, C_t^3)$. The functional form of $U(\cdot)$ is not generation specific. Individuals aim at expected utility maximization. We assume that each individual has an exogenous income of $A > 0$ in every period.

In its lifetime each generation signs two intergenerational social contracts, one with its parents and one with its children. The time structure of the model is illustrated by Figure 1:

< Insert Figure 1 about here >

At the beginning of period t generations t (which is young then) and t-1 (which lives in its working period) sign a *social contract* Γ_{t-1}. This social contract specifies a payment $R_{t-1} \in \Re$ and an amount of intergenerational trade $q_{t-1} \in Q \subseteq [0,1]$ both to be carried out in period t+1. Q denotes the set of admissable intergenerational trades. In a minute we go into further detail.

After agreeing on a contract, but still in period t, generations t and t-1 carry out specific investments e_t and k_{t-1}, respectively. These investments influence the benefits from intergenerational trade. The investments e_t of the younger generation t can be interpreted as an investment in human capital, say education. The investments k_{t-1}, carried out by generation t-1 in its working period, can be understood as the accumulation of the economy's physical capital stock. Investments cause costs $h_1(e_t)$ and $h_2(k_{t-1})$ to the investors. These cost are sunk immediately after the investments have been carried out (i.e., in period t). Apart from investments, generations t and t-1 decide in t on their savings S_t^1 and S_{t-1}^2, i.e. the (positive or negative) amounts of wealth they want to transfer from youth to working age and from working age to retirement, respectively. We assume that savings are invested somewhere outside the economy and bear a time-invariant interest rate $r \geq 0$. Economic prospects not only depend on the investments in human and non-human wealth and on savings, but also on a manifold of other aspects. We summarize them in random variables θ_τ which have a support in some probability space Θ. The time subscript τ indicates that Nature will draw the realizations between periods $\tau+1$ and $\tau+2$ (sorry for that, but anything else would be even more confusing).

Hence, at some date between periods t and t+1, Nature draws a realization of θ_{t-1}. Then both generations t and t-1 know $\sigma_{t-1} := (k_{t-1}, e_t, \theta_{t-1}) \in \Sigma := [0, \overline{K}] \times [0, \overline{E}] \times \Theta$. They now can costlessly renegotiate their initial contract Γ_{t-1} and agree on a new contract, say $\tilde{\Gamma}_{t-1}$. The renegotiation game will be explained in Section 3.2.

Intergenerational trade between generations t and t-1 takes place at the beginning of period t+1 and is based on the terms of the renegotiated contract $\tilde{\Gamma}_{t-1}$. Then generation t has entered its working period and generation t-1 has reached its old age. Denoting the trade level by q_{t-1}, trade yields (possibly negative) benefits of $v^{t-1} = V(q_{t-1}, k_{t-1}, e_t, \theta_{t-1})$ for generation t-1 and of $w^t = W(q_{t-1}, k_{t-1}, e_t, \theta_{t-1})$ for generation t. Generation t pays the (renegotiated) pension R_{t-1} to its parent's generation, which dies at the end of that period.

At the beginning of period t+1, generations t (which has entered its working period) and t+1 sign a contract Γ_t in the same way as did the generations t-1 and t one period before. As well, they decide on k_t, S_t^2 and e_{t+1}, S_{t+1}^1, respectively.

The same pattern continues as time goes by. The model is common knowledge to all generations. All generations are essentially identical. Consumption for generation t in the three periods of its life amounts to:

(1) $$C_t^1 = A - h_1(e_t) - S_t^1;$$

(2) $$C_t^2 = A + (1+r)\cdot S_t^1 - R_{t-1} + W(q_{t-1}, k_{t-1}, e_t, \theta_{t-1}) - h_2(k_t) - S_t^2;$$

(3) $$C_t^3 = A + (1+r)\cdot S_t^2 + R_t + V(q_t, k_t, e_{t+1}, \theta_t).$$

Throughout the paper the following assumptions will be maintained:

(A1) $0 < \overline{K}, \overline{E} < \infty$. The functions h_1 and h_2 are C^2, strictly increasing and convex in e_t and k_t, respectively. They are normalized such that $h_1(0) = h_2(0) = 0$.

(A2) For all t, all $q_{t-1} \in Q$ and $\theta_{t-1} \in \Theta$:
V is C^2 in (k,e), strictly increasing and strictly concave in k and concave in e.
W is C^2 in (k,e), strictly increasing and strictly concave in e and concave in k.

(A3) The function U is C^2 in (C_t^1, C_t^2, C_t^3), strictly increasing in each argument and concave (weak risk aversion). It satisfies the Inada conditions.

(A4) For all t, the distributions G_t of θ_t are independent of e_τ and k_τ for all τ.

Assumptions (A1) to (A3) are primarily technical. They ensure the existence of interior solutions to optimization problems discussed in the sequel. Assumption (A4) says that the generations' investment behaviour does not affect the distribution of stochastic disturbances that hit the economy. This especially implies that Nature is not a channel for investment-induced externalities in the contractual relationship of two generations. Furthermore, moral-hazard effects are excluded. (A4) may be questioned on empirical grounds.

At this point we do not make any assumptions concerning the serial (in)dependence of the θ_t. This will turn out to be of crucial importance (see Section 5).

The contractual relationship between two generations has some specific properties (without loss of generality we only consider generations t and t-1; all other contracts have the same characteristics).

The social contract Γ_{t-1} between generations t-1 and t is signed at date t. We assume that the investments in education and technology, e_t and k_{t-1}, are sufficiently complex that they cannot be contracted *ex ante*. The costs h_t^1 and h_{t-1}^2 are sunk after investment efforts have been undertaken. *Ex post*, e_t and k_{t-1} are verifiable. The random vector θ_{t-1}, too, is assumed to be sufficiently complex such that state-contingent contracts cannot be designed *ex ante*. Contracts can therefore only specify intergenerational terms of trade.

Note that intergenerational trade will only take place if both generations *deliberately* carry out their contractual duties. If at least one of the generations does not want the deal to be carried out (which has to be sharply distinguished from not underwriting the contract), the deal fails; this is the no-trade case. The crucial distinction to be drawn now is whether the generation which causes the failure of the trade can be assigned the responsibility for this event or not:

- In the latter case only *at-will contracts* are feasible which implies that the initial and the renegotiated contracts only can specifiy *one* (possibly negative) payment R_{t-1}^0 from generation t to generation t-1 for the case that at least one of the two generations does for whatever reason not want to carry out the trade (q_{t-1}, R_{t-1}) and the no-trade situation q_{t-1}^0 occurs. An intergenerational contract then is given by $\Gamma_{t-1} = \big((q_{t-1}, R_{t-1}), (q_{t-1}^0, R_{t-1}^0)\big)$. This is the "canonical" case discussed by Hart/ Moore (1988).
- In the first case, "breach penalties" to individual generations become feasible. Hence, the contract can specify default points contingent on the compliance of each of the contracting parties (see Aghion/Dewatripont/Rey (1994)). A specific form of such contracts are option contracts as discussed in Nöldeke/Schmidt (1995).

Assigning the reponsibility for the no-trade event to one of two generations and sentencing generations to breach penalties does not only sound rather strange, but would require a very stark institutional framework. As we do not think this would fit well into our contractarian framework, we ignore this case. This is, however, far from innocuous for the results because we considerably limit the space of feasible social contracts.

Following the literature, we model renegotiation as a bargaining game (see Section 4.3). The payoffs specified by the initial contract Γ_{t-1} are the disagreement point for the renegotiation between generations t and t-1. We denote the renegotiated contract by $\tilde{\Gamma}_{t-1}$.

Renegotiation between generations t and t-1 leads to *ex post efficiency* if under the renegotiated contract $\tilde{\Gamma}_{t-1}$ the amount of intergenerational trade q_{t-1}^* is chosen such that

$$q_{t-1}^*(\sigma_{t-1}) \in \arg\max_{q \in Q} \big(W(q, \sigma_{t-1}) + V(q, \sigma_{t-1})\big).$$

Under *ex post efficient* renegotiation the generations reap the maximum gains of intergenerational trade attainable if the state of the world is given by $\sigma_{t-1} \in \Sigma$ and feasible trades are given by the set Q.

4. The Renegotiation Game and Efficiency

4.1 Preliminaries

This section studies the renegotiation game between two generations t and t-1.

For t, t' with $t - t' \in \{0,1\}$, $\sigma_{t'} \in \Sigma$ and a contract $\Gamma_{t'}$ define

(4) $$P^t(\Gamma_{t'}, \sigma_{t'}) := \begin{cases} R_{t'} + V(q_{t'}, \sigma_{t'}) & \text{if } t = t' \\ -R_{t'} + W(q_{t'}, \sigma_{t'}) & \text{if } t - 1 = t' \end{cases}$$

as the net payoff which accrues to generation t if the contract $\Gamma_{t'}$ is applied in state $\sigma_{t'}$ (i.e., after e_t and $k_{t'}$ have been sunk and $\theta_{t'}$ has been revealed), after the renegotiation game has been played and after trade decisions have been made. Now consider the situations for generations t and t-1 before the renegotiation of their contract Γ_{t-1} starts (i.e., between periods t and t+1):

a) Generation t-1's life is almost over, C^1_{t-1} and C^2_{t-1} have already been realized as well as S^2_{t-1} and k_{t-1} have been fixed. Only has $P^{t-1}(\Gamma_{t-1}, \sigma_{t-1})$ to be determined. (Lifetime) Utility for generation t-1 amounts to

(5) $$\hat{U}^{t-1}(P^{t-1}(\Gamma_{t-1}, \overline{\sigma}_{t-1})) := U\left(\overline{C}^1_{t-1}, \overline{C}^2_{t-1}, A + (1+r)\overline{S}^2_t + P^{t-1}(\Gamma_{t-1}, \overline{\sigma}_{t-1})\right)$$

which due to (A3) strictly increases in $P^{t-1}(\Gamma_{t-1}, \sigma_{t-1})$. Hence, the only aim of generation t-1 is to gain as much as possible from renegotiation.

b) For generation t, C^1_t, S^1_t and e_t are predetermined. But generation t knows that it will write a contract Γ_t with its children and, after that, must decide on k_t and S^2_t. As the time structure of the model and the probability distributions of all future variables are known to generation t, it can (at least in principle) anticipate both the contract Γ_t and its renegotiation. The problem for generation t in period $t+1$ reads:

$$(6)\ \max_{S_t^2, k_t} E_{\theta_t}\left[U\left(\overline{C_t^1}, A+\overline{(1+r)S_t^1}+P^t(\Gamma_{t-1},\overline{\sigma}_{t-1})-S_t^2-h_2(k_t), A+(1+r)S_t^2+P^t(\Gamma_t,k_t,\hat{e}_{t-1},\theta_t)\right)\right],$$

where barred variables are predetermined in $t+1$. The expectation is taken with respect to the distribution of θ_t. By $\hat{e}_{t+1}$ we indicate the optimal investment of its children's generation $t+1$ as seen from the viewpoint of generation t. We make the following assumption:

(A5) When deciding on their investment and savings levels, all generations take the behaviour of all other generations as given. I.e.:

$$\frac{\partial \xi_t}{\partial \eta_\tau} \equiv 0 \qquad \text{for } \xi_\tau, \eta_\tau = e_\tau, k_\tau, S_\tau^1, S_\tau^2 \text{ and for all } t \neq \tau.$$

(A5) says that no generation expects any other generation to react on its own decisions. It is the usual Nash assumption which excludes all conjectural expectations from the analysis.

If we assume existence and uniqueness of an interior solution to the utility maximization problem (6), the optimum values of k_t and S_t^2 satisfy the conditions:

$$(7) \qquad E_{\theta_t}\left[-U_2^t \cdot h_2'(k_t) + U_3^t \cdot P_k^t(\Gamma_t, \sigma_t)\right] = 0,$$

$$(8) \qquad E_{\theta_t}\left[-U_2^t + (1+r) \cdot U_3^t\right] = 0,$$

where $U_n^t := \dfrac{\partial U(C_t^1, C_t^2, C_t^3)}{\partial C^n}$ for $n = 1, 2, 3$ and

$$P_k^t(\Gamma_t, \sigma_t) := \frac{d\, P^t(\Gamma_t, \sigma_t)}{d\, k_t}$$

represents the marginal change in the net payoff of a contract Γ_t to generation t if k_t marginally changes.[8] $P_k^t(\Gamma_t, \sigma_t)$ may be interpreted as the marginal return on an investment k_t under contract terms Γ_t. Equations (7) and (8) can be combined to yield:

$$(9) \qquad h_2'(k_t) = \frac{E_{\theta_t}\left[P_k^t \cdot U_3^t\right]}{(1+r) \cdot E_{\theta_t}\left[U_3^t\right]}.$$

This expression has a familiar structure. The RHS represents the expected marginal utility of an investment k_t relative to the expected marginal utility of savings. The LHS represents the marginal cost of k_t relative to the relative marginal cost of savings which are unity.

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[8] We thus assume that, for any feasible contract and realization of the random variables, net payoffs are differentiable in investment levels.

In order to induce a positive k_t in (9), P_k^t must be strictly positive over some subset of the support of θ_t. This simply means that relational investments must pay a positive return.

Denote the solution to (7) and (8) by $(k_t^*, S_t^{2^*})$ which, among others, depends on $P^t(\Gamma_{t-1}, \sigma_{t-1})$ which is random. Inserting $(k_t^*, S_t^{2^*})$ into the expected utility function (6) yields a value function (as seen by generation t between dates t and t+1) which we denote by

$$\begin{aligned}(10)\quad &\widetilde{U}^t\left(P^t(\Gamma_{t-1}, \overline{\sigma}_{t-1})\right)\\ &:= E_{\theta_t}\left[U\left(\overline{C}_t^1,\, A+(1+r)\overline{S}_t^1 + P^t(\Gamma_{t-1}, \overline{\sigma}_{t-1}) - S_t^{2^*} - h_2(k_t^*),\, A+(1+r)S_t^{2^*} + P^t(\Gamma_t, k_t^*, \hat{e}_{t-1}, \theta_t)\right)\right].\end{aligned}$$

Using the Nash assumption (A5) and the envelope theorem we know that

$$(11)\qquad \frac{\partial \widetilde{U}^t}{\partial P^t(\Gamma_{t-1}, \overline{\sigma}_{t-1})} = E_{\theta_t}\left[U_2^t\right] > 0.$$

Thus, generation t strictly prefers a higher contractual payoff to a lower one. As a result, this is not very surprising. Note however, that from (11) and (5) we can isolate the bilateral renegotiation games between any two generations from the rest of the analysis. Renegotiation can be analysed as if it were the only thing to happen in our model, without caring about reactions and repercussions from other components of the model.

4.2 Pareto-efficient Renegotiation

Lemma 1: Pareto-efficient renegotiation between generations t-1 and t requires:

a) The contract induces ex post efficiency in intergenerational trade.

b) There is some $\lambda \in (0,1)$ such that

$$\lambda \cdot U_3^{t-1} = (1-\lambda)\cdot(1+r)\cdot E_{\theta_t}\left[U_3^t\right].$$

Proof: By definition, for all $\sigma \in \Sigma$ and all contracts Γ_{t-1}, we have

$$P^t(\Gamma_{t-1}, \sigma) + P^{t-1}(\Gamma_{t-1}, \sigma) = W(q, \sigma) + V(q, \sigma)$$

where q denotes the trade level which emerges under Γ_{t-1}. Renegotiation is pareto-efficient if the resulting contract $\widetilde{\Gamma}_{t-1}$ induces payments $\widetilde{P}^t := P^t(\widetilde{\Gamma}_{t-1}, \sigma)$ and $\widetilde{P}^{t-1} = P^{t-1}(\widetilde{\Gamma}_{t-1}, \sigma)$ and corresponding at-will trade levels such that for some $\lambda \in (0,1)$

$$\lambda \cdot \hat{U}^{t-1}(\widetilde{P}^{t-1}) + (1-\lambda)\cdot \widetilde{U}^t(\widetilde{P}^t)$$

is maximized subject to

$$\widetilde{P}^t + \widetilde{P}^{t-1} = W(q, \sigma) + V(q, \sigma).$$

a) As $\hat{U}^{t-1}$ and $\tilde{U}^{t}$ are both strictly increasing in P^{t-1} and P^{t} respectively, a contract which induces a trade $q' \notin \arg\max_{q \in Q} \left(W(q,\sigma_{t-1}) + V(q,\sigma_{t-1})\right)$ can be Pareto-improved.

b) The Lagrange problem associated with Pareto-efficiency yields as a necessary FOC:

$$\lambda \cdot \frac{\partial \hat{U}^{t-1}(\tilde{P}^{t-1})}{\partial P} = (1-\lambda) \cdot \frac{\partial \tilde{U}^{t}(\tilde{P}^{t})}{\partial P}.$$

Applying (5) yields $\frac{\partial \hat{U}^{t-1}(\tilde{P}^{t-1})}{\partial P} = U_3^{t-1}$. Using (11) and (8) yields

$$\frac{\partial \tilde{U}^{t}(\tilde{P}^{t})}{\partial P} = E_{\theta_t}\left[U_2^t\right] = (1+r) \cdot E_{\theta_t}\left[U_3^t\right].$$

This immediately leads to the assertion.

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4.3 The renegotiation game

Now let us consider the renegotiation game. Suppose the initial contract Γ_{t-1} specified an intergenerational trade level q_{t-1} and pension payments R_{t-1} and R_{t-1}^0 for the trade case and for the no-trade case $q_{t-1}^0 = 0$, respectively. Now some $\sigma_{t-1} \in \Sigma$ has been realized and renegotiation can start. Denote by

$$q(\Gamma_{t-1},\sigma_{t-1}) \in \{0, q_{t-1}\}$$

the trade level that would emerge if the initial contract were not renegotiated. I.e.,

$$q(\Gamma_{t-1},\sigma_{t-1}) = q_{t-1} \text{ if and only if}$$
$$\left[V(q_{t-1},\sigma_{t-1}) + R_{t-1} \geq V(0,\sigma_{t-1}) + R_{t-1}^0 \;\;\&\;\; W(q_{t-1},\sigma_{t-1}) - R_{t-1} \geq W(0,\sigma_{t-1}) - R_{t-1}^0\right].$$

Denote by $p^{t-1}(\Gamma_{t-1},\sigma_{t-1})$ and $p^{t}(\Gamma_{t-1},\sigma_{t-1})$ the net payoffs of the initial contract at the trade level $q(\Gamma_{t-1},\sigma_{t-1})$. Define $\mu(\Gamma_{t-1},\sigma_{t-1}) = \left(\hat{U}^{t-1}(p^{t-1}(\Gamma_{t-1},\sigma_{t-1})), \tilde{U}^{t}(p^{t}(\Gamma_{t-1},\sigma_{t-1}))\right)$. As earlier,

$$q_{t-1}^*(\sigma_{t-1}) \in \arg\max_{q' \in Q} \left(W(q',\sigma_{t-1}) + V(q',\sigma_{t-1})\right)$$

denotes an *ex post* efficient trade level. Assume that $q_{t-1}^*(\sigma_{t-1})$ is unique. Define

$$M(\sigma_{t-1}) := \left\{\left(\hat{U}^{t-1}(x^{t-1}), \tilde{U}^{t}(x^{t})\right) \,\middle|\, x^{t-1} + x^{t} \leq W(q_{t-1}^*(\sigma_{t-1}),\sigma_{t-1}) + V(q_{t-1}^*(\sigma_{t-1}),\sigma_{t-1})\right\}.$$

Then the renegotiation game can be written as a pair

$$\Phi(\Gamma_{t-1},\sigma_{t-1}) = \left(M(\sigma_{t-1}), \mu(\Gamma_{t-1},\sigma_{t-1})\right)$$

where M is the utility space and μ the disagreement point. By the concavity and continuity assumptions on the utility functions, M is convex and comprehensive. Clearly, $\mu(\Gamma_{t-1}, \sigma_{t-1}) \in M(\sigma_{t-1})$. A solution to this standard-type two-player bargaining problem will be written as a function

$$F\colon \Phi \mapsto M(\sigma_{t-1})$$

which determines for any given initial contract Γ_{t-1} and for any realization $\sigma_{t-1} \in \Sigma$ the outcome of the renegotiation game.

We do not specify the bargaining procedure any further. We, however, assume, that the following three reasonable and rather innocuous axioms concerning the renegotiation game hold:[9]

(R1) F is pareto-efficient (see Lemma 1).
(R2) F is individually rational, i.e. $F(\Phi) \geq \mu(\Gamma_{t-1}, \sigma_{t-1})$.
(R3) F is weakly monotonic with respect to the disagreement point, i.e.:

$$\mu' \geq \mu \Rightarrow F(M, \mu') \geq F(M, \mu).$$

These requirements encompass a rather wide range of possible bargaining outcomes, including the dictatorial solution, egalitarianism, the Nash and the Kalai-Smorodinsky solutions. Some evident properties are summarized in

Proposition 1:
Suppose, (R1)-(R3) hold.

a) If $q^*(\sigma_{t-1}) \in \{0, q_{t-1}\}$, then $\tilde{\Gamma}_{t-1} = \Gamma_{t-1}$ and $F(\Phi) = \mu(\Gamma_{t-1}, \sigma_{t-1})$.

b) If $q^*(\sigma_{t-1}) \neq q(\Gamma_{t-1}, \sigma_{t-1})$, the initial contract will be renegotiated. Especially:

(i) If generation t is dictatorial (i.e., has full bargaining power), then:
$\tilde{P}^{t-1} = p^{t-1}$ and $\tilde{P}^{t} = \left[W(q^*(\sigma_{t-1}), \sigma_{t-1}) + V(q^*(\sigma_{t-1}), \sigma_{t-1}) - p^{t-1}\right]$.

(ii) If generation t-1 is dictatorial, then:
$\tilde{P}^{t} = p^{t}$ and $\tilde{P}^{t-1} = \left[W(q^*(\sigma_{t-1}), \sigma_{t-1}) + V(q^*(\sigma_{t-1}), \sigma_{t-1}) - p^{t}\right]$.

Proposition 1 merely restates standard results from bargaining theory:

- If the *ex post* efficient trade level is dealt with in the initial contract and R has to be chosen such as to induce the efficient level of trade, the contract will not be renegotiated (part a)).
- If there are gains from renegotiation, they will be reaped (part b)).

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9 Vector inequalities are meant to comprise equality.

- If one party of the renegotiation game has all the bargaining power it will fully "exploit" the other party by throwing it down to its fallback position from the initial contract. The "dictatorial" party will allocate the total surplus from trade to itself.

In the Appendix we provide two more specific examples of intergenerational contracting. The first assumes that the trade objects are property rights to the economy's capital stock and is essentially a reinterpretation of the Hart/Moore (1988) model. The second example demonstrates how bargaining on the age of retirement could be modelled in our setting.

5. Investment and Savings Decisions

We so far restricted our attention to the renegotiation game. We now turn to the investment and savings decisions (e_t, S_t^1) and (k_{t-1}, S_{t-1}^2) which two generations t and t-1 have to make after writing the initial contract. When doing so, the generations take into account that the initial contract may be renegotiated after the uncertainty has resolved and that the outcome of the renegotiation will be determined by a commonly known bargaining solution F.

5.1 Individual behaviour

5.1.1 The older generation

We know already from Section 3 that the older generation (i.e., generation t-1) fixes (k_{t-1}, S_{t-1}^2) such that

(7') $$E_{\theta_{t-1}}\left[-U_2^{t-1} \cdot h_2'(k_{t-1}) + U_3^{t-1} \cdot \widetilde{P}_k^{t-1}(\Gamma_{t-1}, \sigma_{t-1})\right] = 0$$

and

(8') $$E_{\theta_{t-1}}\left[-U_2^{t-1} + (1+r) \cdot U_3^{t-1}\right] = 0$$

simultaneously hold. In $\widetilde{P}_k^{t-1}(\Gamma_{t-1}, \sigma_{t-1}) := \dfrac{d\, P^{t-1}(\widetilde{\Gamma}_{t-1}, \sigma_{t-1})}{d\, k_{t-1}}$ the outcome of the renegotiation game (under a given rule F) is anticipated.

5.1.2 The younger generation

The younger generation (i.e., generation t) chooses (e_t, S_t^1) as to maximize

$$E_{\theta_{t-1},\theta_t}\left[U\left(A-h_1(e_t)-S_t^1,\ A+(1+r)S_t^1+P^t(\tilde{\Gamma}_{t-1},\sigma_{t-1})-h_2(k_t^*)-S_t^{2^*},C_t^{3^*}\right)\right],$$

where the starred variables denote future decisions. Generation t assumes that these decisions will be optimally taken, i.e., that they satisfy (7) and (8) for parametrically given C_t^1 and $A+(1+r)S_t^1+\tilde{P}^t$. (Technically, generation t uses backward induction to solve its dynamic optimization problem.) Under the Nash assumption (A5) the FOC for this problem are:

(13a)
$$E_{\theta_t,\theta_{t-1}}\Big[-U_1^t\cdot h_t'(e_t)+U_2^t\cdot \tilde{P}_e^t(\tilde{\Gamma}_{t-1},\sigma_{t-1}) +U_2^t\cdot\left(-h_2'(k_t^*)\cdot\frac{\partial k_t^*}{\partial e_t}-\frac{\partial S_t^{2^*}}{\partial e_t}\right)+U_3^t\cdot\left((1+r)\cdot\frac{\partial S_t^{2^*}}{\partial e_t}+\frac{\partial \tilde{P}^t(\tilde{\Gamma}_t,\sigma_t)}{\partial e_t}\right)\Big]=0$$

(13b)
$$E_{\theta_t,\theta_{t-1}}\Big[-U_1^t+U_2^t\cdot(1+r) +U_2^t\cdot\left(-h_2'(k_t^*)\cdot\frac{\partial k_t^*}{\partial S_t^1}-\frac{\partial S_t^{2^*}}{\partial S_t^1}\right)+U_3^t\cdot\left((1+r)\cdot\frac{\partial S_t^{2^*}}{\partial S_t^1}+\frac{\partial \tilde{P}^t(\tilde{\Gamma}_t,\sigma_t)}{\partial S_t^1}\right)\Big]=0$$

From Section 4.1 we know that the payoff after renegotiation only depends on the initial contract, the investment decisions connected with this (and only this) contract and the state of the world in the renegotiation game. Therefore:

(14a)
$$\frac{\partial \tilde{P}^t(\tilde{\Gamma}_t,\sigma_t)}{\partial e_t}=\frac{\partial \tilde{P}^t(\tilde{\Gamma}_t,k_t^*,e_{t-1},\theta_t)}{\partial k_t}\cdot\frac{\partial k_t^*}{\partial e_t}$$

and

(14b)
$$\frac{\partial \tilde{P}^t(\tilde{\Gamma}_t,\sigma_t)}{\partial S_t^1}=\frac{\partial \tilde{P}^t(\tilde{\Gamma}_t,k_t^*,e_{t-1},\theta_t)}{\partial k_t}\cdot\frac{\partial k_t^*}{\partial S_t^1}.$$

Recall that the decisions on k_t^* and S_t^2 have to be made before θ_t has been revealed. Therefore they cannot depend on θ_t. The same must hold for the partial derivatives $\frac{\partial k_t^*}{\partial e_t}$ and $\frac{\partial k_t^*}{\partial S_t^1}$ in (14). Generation t knows that for any state of the world θ_{t-1} which will have been revealed between t and t+1 it will in t+1 choose k_t and S_t^2 such that conditions (7) and (8) will hold. To simplify the analysis we make the following assumption:

(A6) For all t,t' with $t\neq t'$ the random variables θ_t and $\theta_{t'}$ are stochastically independent.

We can now characterize the optimal investment behaviour of the younger generation:

Proposition 2:

Assume (A1) through (A6) hold. Then generation t chooses e_t and S_t^1 such that

(15a) $$E_{\theta_t,\theta_{t-1}}\left[-U_1^t \cdot h_1'(e_t) + U_2^t \cdot \widetilde{P}_e'(\widetilde{\Gamma}_{t-1},\sigma_{t-1})\right] = 0$$

and

(15b) $$E_{\theta_t,\theta_{t-1}}\left[-U_1^t + U_2^t \cdot (1+r)\right] = 0.$$

simultaneously hold.

Proof: Consider equation (13a) which by using (14a) and (14b) can be rewritten as:

$$E_{\theta_t,\theta_{t-1}}\left[U_1^t \cdot h_1'(e_t) - U_2^t \cdot \widetilde{P}_e'(\widetilde{\Gamma}_{t-1},\sigma_{t-1})\right]$$

$$= E_{\theta_t,\theta_{t-1}}\left[\frac{\partial k_t^*}{\partial e_t} \cdot \left(-U_2^t \cdot h_2'(k_t^*) + U_3^t \cdot \widetilde{P}_k'(\widetilde{\Gamma}_t,\sigma_t)\right)\right] + E_{\theta_t,\theta_{t-1}}\left[\frac{\partial S_t^{2^*}}{\partial e_t} \cdot \left(-U_2^t + (1+r) \cdot U_3^t\right)\right]$$

Each of the two summands on the RHS has the form $E_{\theta_t,\theta_{t-1}}[\alpha(\theta_{t-1}) \cdot \beta(\theta_t,\theta_{t-1})]$ where $\alpha(\cdot)$ and $\beta(\cdot)$ are appropriately defined functions. Under (A6) (i.e., with θ_t and θ_{t-1} independent) these expressions can be calculated as follows:

$$E_{\theta_t,\theta_{t-1}}[\alpha(\theta_{t-1}) \cdot \beta(\theta_t,\theta_{t-1})] = \iint_{\Theta\times\Theta}[\alpha(\theta_{t-1}) \cdot \beta(\theta_t,\theta_{t-1})]\, dG(\theta_{t-1})\, dG(\theta_t)$$

$$= \int_\Theta \alpha(\theta_{t-1}) \cdot \left(\int_\Theta \beta(\theta_t,\theta_{t-1})\, dG(\theta_t)\right) dG(\theta_{t-1}) = \int_\Theta \alpha(\theta_{t-1}) \cdot E_{\theta_t}(\beta(\cdot,\theta_{t-1}))\, dG(\theta_{t-1})$$

(see Karr (1993, Theorem 4.29)). From (7) and (8) we know however that $E_{\theta_t}[\beta(\cdot,\theta_{t-1})] = 0$ for all $\theta_{t-1} \in \Theta$. This makes the integrals and hence the RHS zero, too. Eq. (14a) follows immediately. The same procedure applied to (13b) yields (15b).

□

Proposition 2 heavily depends on assumption (A6). Under (A6), we can simply "insert" (7) and (8) into (13) and thus separate the decisions of different life periods. This would not be possible if θ_t and θ_{t-1} were stochastically dependent (and their joint distribution not multiplicatively separable). Clearly, (A6) is very simplifying: the random variables θ_t represent "shocks" in the macroeconomic sense which empirically are not serially independent.

Note from (14a) and (14b) that all generations are interconnected: Although the investment level e_t at first sight only influences the intergenerational contract Γ_{t-1} between generations t and t-1, the payoffs from that contract accrue before the contract Γ_t between generations t and t+1 is signed. Hence, capital investments k_t are influenced by earlier education investments e_t of the same generation and, via the renegotiation of the contract Γ_t, also on the capital investment k_{t-1} of its parents and on the realizations of past stochastic shocks.

Finally, note that serial independence of the θ_t is not necessary to generate separability in the decisions in the sense of Proposition 2. If utility functions are additively separable in the period consumtion levels it is easily checked that even without (A6) generations can view their bargaining as a fully bilateral affair (a familiar example for this would be $U(C_t^1, C_t^2, C_t^3) = u(C_t^1) + \delta \cdot u(C_t^2) + \delta^2 \cdot u(C_t^3)$ with $0 < \delta \le 1$).

5.2 Efficiency properties

In order to be pareto-efficient, investment and savings levels $e_t, S_t^1, k_{t-1}, S_t^2$ should for some $\lambda \in]0,1[$ solve the maximization problem

$$\max_{e_t, S_t^1, k_{t-1}, S_{t-1}^2} \lambda \cdot E_{\theta_{t-1},\theta_t}\left[U(C_t^1, C_t^2, \hat{C}_t^3)\right] + (1-\lambda)\cdot E_{\theta_{t-1}}\left[U(\overline{C}_{t-1}^1, C_{t-1}^2, C_{t-1}^3)\right],$$

where the C_τ^j are as defined in (1), (2), and (3) and the outcome of the renegotiation game is incorporated in the analysis.[9] Hat values indicate that the respective variable is assumed to take a pareto-efficient level. The FOC of the social planning problem are:

$$\begin{aligned}(16a)\quad & \lambda \cdot E_{\theta_{t-1},\theta_t}\Big[-h_1'(e_t)\cdot U_t^1 + U_2^t \cdot \widetilde{P}_e^t(\Gamma_{t-1},\sigma_{t-1}) + \frac{\partial \hat{k}_t}{\partial e_t}\cdot\left(-h_2'(\hat{k}_t)\cdot U_2^t + U_3^t \cdot \widetilde{P}_k^t(\Gamma_t,\hat{\sigma}_t)\right) \\ & + \frac{\partial \hat{S}_t^2}{\partial e_t}\cdot\left(-U_2^t + (1+r)\cdot U_3^t\right)\Big] + (1-\lambda)\cdot E_{\theta_{t-1}}\left[U_3^{t-1}\cdot \widetilde{P}_e^{t-1}(\Gamma_{t-1},\sigma_{t-1})\right] = 0\end{aligned}$$

$$\begin{aligned}(16b)\quad & \lambda \cdot E_{\theta_{t-1},\theta_t}\Big[-U_t^1 + U_2^t\cdot(1+r) + \frac{\partial \hat{k}_t}{\partial S_t^1}\cdot\left(-h_2'(\hat{k}_t)\cdot U_2^t + U_3^t \cdot \widetilde{P}_k^t(\Gamma_t,\hat{\sigma}_t)\right) \\ & + \frac{\partial \hat{S}_t^2}{\partial S_t^1}\cdot\left(-U_2^t + (1+r)\cdot U_3^t\right)\Big] = 0\end{aligned}$$

$$\begin{aligned}(16c)\quad & \lambda \cdot E_{\theta_{t-1},\theta_t}\Big[U_2^t \cdot \widetilde{P}_k^t(\Gamma_{t-1},\sigma_{t-1}) + \frac{\partial \hat{k}_t}{\partial k_{t-1}}\cdot\left(-h_2'(\hat{k}_t)\cdot U_2^t + U_3^t\cdot \widetilde{P}_k^t(\Gamma_t,\hat{\sigma}_t)\right) \\ & + \frac{\partial \hat{S}_t^2}{\partial k_{t-1}}\cdot\left(-U_2^t + (1+r)\cdot U_3^t\right)\Big] \\ & + (1-\lambda)\cdot E_{\theta_{t-1}}\left[-U_2^{t-1}\cdot h_2'(k_{t-1}) + U_3^{t-1}\cdot \widetilde{P}_k^{t-1}(\Gamma_{t-1},\sigma_{t-1})\right] = 0\end{aligned}$$

---

9 Thus, we take the bargaining procedure as presented in Section 3.2 as *given* and do not analyse Pareto-efficient bargaining as defined by Lemma 1.

$$\text{(16d)}\quad \lambda \cdot E_{\theta_{t-1},\theta_t}\left[\frac{\partial \hat{k}_t}{\partial S_{t-1}^2}\cdot\left(-h_2'(\hat{k}_t)\cdot U_2^t + U_3^t\cdot \tilde{P}_k^t(\Gamma_t,\hat{\sigma}_t)\right) + \frac{\partial \hat{S}_t^2}{\partial S_{t-1}^2}\cdot\left(-U_2^t+(1+r)\cdot U_3^t\right)\right] + (1-\lambda)\cdot E_{\theta_{t-1}}\left[-U_2^{t-1}+U_3^{t-1}\cdot(1+r)\right] = 0$$

(16a) to (16d) exhibit that a social planner who aims at fixing investment and savings levels efficiently faces quite a difficult task. It therefore seems interesting to investigate whether the generations themselves reach an efficient allocation without the interference of some benevolent and omniscient agency (whomever this represents in our social contract setting). We thus insert the conditions obtained in the previous section (i.e., (7), (8) and (14)) into (16).

First check that for the same reasons as in the proof of Proposition 2 all terms connected with partial derivatives of $\hat{k}_t$ and $\hat{S}_t^2$ can be ignored if (A6) holds. The respective expected values are zero due to (8), (14) and the independence of θ_t and θ_{t-1}. Applying (8) and (14) a second time yields that (16b) and (16d) are satisfied by individual behaviour and that the LHS of (16a) and (16c) reduce to

$$\text{(17a)}\qquad (1-\lambda)\cdot E_{\theta_{t-1}}\left[U_3^{t-1}\cdot \tilde{P}_e^{t-1}(\Gamma_{t-1},\sigma_{t-1})\right]$$

and

$$\text{(17b)}\qquad \lambda\cdot E_{\theta_{t-1},\theta_t}\left[U_2^t\cdot \tilde{P}_k^t(\Gamma_{t-1},\sigma_{t-1})\right],$$

respectively. Hence, we obtain

Proposition 3:

Let (A1) to (A6) be satisfied. Denote by $\hat{e}_t, \hat{k}_{t-1}$ the pareto-efficient investment levels for generations t and t-1 in period t. Denote by e_t^* and k_{t-1}^* the respective individually optimal levels. Then:

$$\operatorname{sgn}\left(\hat{e}_t - e_t^*\right) = \operatorname{sgn}\left(E_{\theta_{t-1}}\left[U_3^{t-1}\cdot \tilde{P}_e^{t-1}(\Gamma_{t-1},\sigma_{t-1})\right]\right)$$

and

$$\operatorname{sgn}\left(\hat{k}_{t-1} - k_{t-1}^*\right) = \operatorname{sgn}\left(E_{\theta_{t-1},\theta_t}\left[U_2^t\cdot \tilde{P}_k^t(\Gamma_{t-1},\sigma_{t-1})\right]\right).$$

The interpretation of Proposition 3 is straightforward: If generation t expects that its investment e_t is to the benefit of generation t-1 via the intergenerational contract (i.e., if (17a) is positive), then it underinvests: $e_t^* < \hat{e}_t$. In the same manner, generation t-1 chooses a suboptimally low investment level $k_{t-1}^* < \hat{k}_{t-1}$ if it expects that the intergenerational contract shifts some of the benefits from the investment to its children. On the converse, if a generation's investment harms the other generation (i.e., (17a) or (17b) are negative), then individual optimization ignores this negative externality and leads to investment levels which are too high. Clearly, if both (17a) and (17b) are zero, no externalities occur and both investment and savings levels are chosen efficiently by the generations.

6. Sources of intergenerational externalities

We now identify the sources of the intergenerational externalities which, according to Proposition 3, lead to under- or over-investment.

6.1 Allocation of the bargaining power to one generation

First, let us consider the dictatorial bargaining solution. Furthermore, we shall concentrate on the externalities generated by generation t and thus on the influence of e_t on the term given in (17a). The arguments for term (17b) are similar. Recall from Proposition 1 that:

- if generation t-1 has the bargaining power:

$$\tilde{P}^{t-1}(\Gamma_{t-1},\sigma_{t-1}) =$$
$$\begin{cases} W(q^*_{t-1}(\sigma_{t-1}),\sigma_{t-1}) + V(q^*_{t-1}(\sigma_{t-1}),\sigma_{t-1}) - [W(q_{t-1},\sigma_{t-1}) - R_{t-1}] & \text{if } q^*_{t-1}(\sigma_{t-1}) \neq 0 \;\; \& \;\; q(\Gamma_{t-1}) = q_{t-1} \\ W(q^*_{t-1}(\sigma_{t-1}),\sigma_{t-1}) + V(q^*_{t-1}(\sigma_{t-1}),\sigma_{t-1}) - [W(0,\sigma_{t-1}) - R^0_{t-1}] & \text{if } q^*_{t-1}(\sigma_{t-1}) \neq 0 \;\; \& \;\; q(\Gamma_{t-1}) = 0 \\ V(0,\sigma_{t-1}) + R^0_{t-1} & \text{if } q^*_{t-1}(\sigma_{t-1}) = 0 \end{cases}$$

(18a)

- if generation t has the bargaining power:

$$\tilde{P}^{t-1}(\tilde{\Gamma}_{t-1},\sigma_{t-1}) = \begin{cases} V(q_{t-1},\sigma_{t-1}) + R_{t-1} & \text{if } q^*_{t-1}(\sigma_{t-1}) \neq 0 \;\; \& \;\; q(\Gamma_{t-1}) = q_{t-1} \\ V(0,\sigma_{t-1}) + R^0_{t-1} & \text{if } q^*_{t-1}(\sigma_{t-1}) \neq 0 \;\; \& \;\; q(\Gamma_{t-1}) = 0 \\ V(0,\sigma_{t-1}) + R^0_{t-1} & \text{if } q^*_{t-1}(\sigma_{t-1}) = 0 \end{cases} \qquad \text{(18b)}$$

Consider the if-clauses in (18): the first condition relates to the ex-post efficient trade level and the second to the trade level that would be carried out in state σ_{t-1} if the initial contract were not modified. Recall that $\sigma_{t-1} = (e_t, k_{t-1}, \theta_{t-1})$.

Then four types of externalities can be identified:

a) There is a *direct externality* because the valuation $V(q_{t-1},\sigma_{t-1})$ that generation t-1 attaches to a trade level q_{t-1} may depend on e_{t-1} via σ_{t-1}. As an example think of V as the present value of the economy's capital stock to generation t-1. The returns to that capital stock change with the skills and abilities of those who work with the capital stock and thus vary positively with the educational efforts e_t of generation t.

b) There is an *optimal trade externality* in the sense that the ex post efficient level of trade q^*_{t-1} depends on the investment decisions of the two generations. Seen ex ante, by changing e_t generation t influences the probability distribution over q^*_{t-1}.[10] This also affects the payoffs to generation t-1. These effects, which are incorporated in the first conditional clauses in (18), have first been identified by Hart/Moore (1988).

c) There is a *surplus externality*. Take the case where generation t-1 has the bargaining power (see (18a)). Then we know that generation t-1 can reap all the surplus from trade and generation t is thrown back to its initial position. The total surplus of trade, however, consists of terms $W(\cdot,\sigma_{t-1})$ which according to Assumption (A2) strictly increase in e_t. If generation t varies e_t, then generation t-1 will be affected by this through the channel of bargaining. Note, however, that the direction of this externality is not clear. Take, e.g., the second line in (18) and assume there are no externalities of types a) or b). Then calculate the payoff effect for generation t-1 when e_t changes as

$$\frac{\partial W(q^*_{t-1},\sigma_{t-1})}{\partial e_t} - \frac{\partial W(q_{t-1},\sigma_{t-1})}{\partial e_t}.$$

The sign of this expression is indeterminate.

d) Finally, there is a *threat point externality*: The ex post optimal level of trade under the initial contract (i.e., whether q_{t-1} or no-trade would occur in state σ_{t-1} if the initial contract Γ_{t-1} were not modified) varies with e_t. These effects, which can be seen in the second of the conditional clauses in (18), are relevant for the fallback positions of the generation which is not endowed with the bargaining power. They determine the extent to which the party with the bargaining power can exploit the other generation.

As it is already difficult to assess the direction of the single externalities, there seems to be hardly any chance for calculating the overall direction (i.e., the sum of a) to d)).

6.2 Non-dictatorial Renegotiation

Unlike under a dictatorial regime non-dictatorial renegotiation games in general do not allocate the whole monetary surplus from renegotiation to one of the parties, but somehow splits it between them. To this end an evaluation of the surplus in utility terms is necessary. Therefore,

10 This happens although the distribution of the random variables do by (A4) not depend on any choice variables.

a fifth type of intergenerational externality occurs: a *bargaining externality.* This can be illustrated if we assume that the renegotiation game is solved by the standard Nash bargaining solution. Given two generations $t-1$ and t, an initial contract Γ_{t-1} and a state of Nature σ_{t-1}, the Nash solution allocates feasible net payments $\tilde{P}^t$ and $\tilde{P}^{t-1}$ to the generations such that

$$\left[\hat{U}^{t-1}(\tilde{P}^{t-1})-\hat{U}^{t-1}(p^{t-1})\right]\cdot\left[\tilde{U}^t(\tilde{P}^t)-\tilde{U}^t(p^t)\right]$$

is maximized. In this product the utility values for generation t-1 depend on $h_2(k_{t-1})$, and those for generation t depend on $h_1(e_t)$ as investment have already been sunk. Therefore the maximizer of the product will typically depend on these investment costs, too. Similar effects occur with bargaining procedures other than the Nash programme. Hence, the payoff of the renegotiation game for each generation is influenced by investment decisions of the other generation via the bargaining solution. The direction of this influence (i.e., wether the externalities are positive or negative) is not clear.

7. Implications and Conclusions

Sections 6.1 and 6.2 identified a number of intergenerational externalities which distort education and capital investments away from their efficient levels. The indirect channel through which they work is the renegotiation of the social contract where investments spill over to the benefit or the harm of the other generation(s). In this section we briefly discuss some preliminary consequences of this observation.

7.1 Optimal PAYG Contracts

We so far have not considered the *initial* social intergenerational contract Γ, but focussed on the renegotiated one $\tilde{\Gamma}$. The initial contract is signed before investments are carried out. In our interpretation it specifies the terms of the intergenerational transfer of property rights with the price component as a PAYG pension to the older generation. The interesting question is whether there exists some type of initial contract (i.e., some PAYG system) - and any rules of the renegotiation game - which avoids the intergenerational externalities just mentionend. This kind of PAYG system would lead to first-best investment levels in all time periods.

At the moment we are not able to answer this question. However, the prospects of finding a first-best PAYG system are rather gloomy, seen against the background of the theoretical literature on incomplete contracts and renegotiation. In their seminal paper Hart/Moore (1988) show that in a bilateral relationship where only the optimal trade externality (item b in Section 6.1) occurs *no first best at-will contract* exists, but that trade partners will underinvest.

As we allow only for at-will contracts this result alone would be sufficient to destroy all hope for finding an efficient PAYG system. If we additionally take into account that the optimal trade externality is just one intergenerational spill-over effect out of a series of five, then we have very good reasons to quit the search before really starting it:

Conjecture: There does not exist any PAYG system in the form of an incomplete social contract with the possibility of renegotiation that leads to first-best education and capital investments.

7.2 Comparing PAYG and Funded Pension Systems

So far we have not discussed the role of savings S_t^1 and S_t^2 in our model. By saving individuals allocate their consumption stream over time and especially determine intertemporal transfers from lifetime periods 1 and 2 to the old age period 3. We can thus interpret savings as a funded pension system with a rate of return r. From Section 5.2 we know that under (A1) to (A6) saving decisions are made efficiently whereas investment decisions are distorted by intergenerational externalities. Of course, intergenerational externalities can and do not play any role for savings as the returns on savings are fully private. This hints at an aspect in the comparison of PAYG and funded pensions systems which to the best of our knowledge so far has not been discussed in the literature:

Conjecture: An advantage of funded pension systems over PAYG systems is that they do not create intergenerational externalities via the renegotiation of a social contract.

In a funded system, generations do not face the risk of partial expropriation of their returns on investment by their parents or children.[11]

7.3 Concluding Remarks and Directions for Further Research

This paper does neither present a full-fledged theory nor a large bunch of results, but is merely a first sketch of an idea that may be helpful in explaining the features and shortcomings of real-

[11] Admittedly, the comparison is not fully fair as we take the interest rate r as given and constant (we assume that savings are invested somewhere outside the economy). In a complete model the interest rate are determined endogenously on the capital markets. This may create channels for intergenerational externalities and hence a fully funded system would be inefficient as well. However, externalities induced by intergenerational bargaining cannot occur.

world PAYG systems. In the current state the paper has stopped short of adressing the really serious problems. To name but a few:

- We largely neglect the equilibrium analysis of the game we present. Formally, our model is an infinitely repeated noncooperative game with finitely-lived overlapping generations of players. While there exists some literature on such games under certainty (see, e.g., Salant (1991) or Kandori (1992)), we are not aware of any analysis under uncertainty.
- We did not fully characterize an "optimal" PAYG renegotiable system (including the design of the initial contract) but only compared some FOC with the decentralized setting.
- Our approach also captures intergenerational risk sharing features which may at least partially justify the notion of an old-age "insurance". These features have to be elaborated more clearly.

Our paper links three branches of economic theory: the theory of PAYG pension systems, the theory of constitutional choice and the incomplete contract approach which recently received great attention in the industrial organization literature. The new element we add to the latter field is the overlapping time structure of our model. From "standard" constitutional choice theory our approach differs by capturing the possibility of renegotiating the social contract. To the literature on public pensions we add a new positive explanation for PAYG systemes which does not rely on altruism and which does not run into the problem of how to bind future generations to the terms of a social contract which they were never asked to agree upon.

Appendix: Two examples of intergenerational contracting

Example 1: Take-it-or-leave-it contracts

Let $Q = [0,1]$ or (which yields the same results) $Q = \{0,1\}$. Consider two generations t and t-1 and assume that:

$$V(q,k_{t-1},e_t,\theta_{t-1}) = (1-q)\cdot f^o(k_{t-1},e_t,\theta_{t-1})$$

and

$$W(q,k_{t-1},e_t,\theta_{t-1}) = q\cdot f^y(k_{t-1},e_t,\theta_{t-1})\,.$$

This may have the following interpretation: q denotes the share of ownership rights to the society's capital stock which is allocated to the younger generation t (for simplicity assume that the total capital stock is normalized to one). f^o and f^y denote the output per unit of capital which can be produced by the older and the younger generations (i.e., by generation t-1 in their old age and generation t in their middle age), respectively.[12] For a given state of the world σ_{t-1} it is straightforward to see that

$$q^*_{t-1}(\sigma_{t-1}) = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \Leftrightarrow f^o(\sigma_{t-1}) \begin{Bmatrix} > \\ \leq \end{Bmatrix} f^y(\sigma_{t-1}).$$

It is efficient to allocate the capital stock completely to the generation with the higher productivity. Assume that when writing their initial contract the generations already anticipate that either full or no transferral will occur ex post and that Γ_{t-1} only specifies two pension payments R_{t-1} and R^0_{t-1} for the cases $q_{t-1} = 1$ and $q_{t-1} = 0$, respectively. In cash, the net payoffs for generation t-1 amount to $f^o + R^0_{t-1}$ in case that $q_{t-1} = 0$ and to R_{t-1} else. The corresponding values for generation t are given by $-R^0_{t-1}$ and $f^y - R_{t-1}$.

Suppose, that F is a dictatorial solution. The following observation is an immediate application of our Proposition 1b and corresponds to Proposition 1 in Hart/Moore (1988). For sake of convenience, we omit the time subscripts.

Fact: a) Suppose, $f^y(\sigma) \geq f^o(\sigma)$ and thus $q^* = 1$.

a1) Suppose further, the initial contract satisfies $f^y(\sigma) \geq R - R^0 \geq f^o(\sigma)$. Equilibrium net payoffs of the renegotiation game are given by

(i) $\tilde{P}^{t-1} = R$ and $\tilde{P}^t = f^y(\sigma) - R$ if generation t proposes the new contract;

(ii) $\tilde{P}^{t-1} = f^y(\sigma) + R$ and $\tilde{P}^t = -R$ if generation t-1 proposes the new contract.

a2) If instead the initial contract satisfies $(R - R^0) \notin \left]f^o(\sigma), f^y(\sigma)\right[$. Then equilibrium payoffs of the renegotiation game are given by

(i) $\tilde{P}^{t-1} = R^0$ and $\tilde{P}^t = f^y(\sigma) - R^0$ if generation t proposes the new contract;

(ii) $\tilde{P}^{t-1} = f^y(\sigma) + R^0$ and $\tilde{P}^t = -R^0$ if generation t-1 proposes the new contract.

b) Suppose, $f^y(\sigma) < f^o(\sigma)$ and thus $q^* = 0$. Then equilibrium payoffs are given by $\tilde{P}^{t-1} = f^o(\sigma) + R^0$ and $\tilde{P}^t = -R^0$.

Fact 1 again shows the extreme feature of the renegotiation game under the dictatorial solution, namely that the generation which is endowed with the bargaining power fully exploits the other generation. Furthermore, as by our specification of the payoffs the no-trade event is the situation where the capital stock is fully owned by the older generation and the decisions for a transfer of property rights to the younger generation have to be agreed upon by both

12 Hence, younger and older workers are imperfect substitutes in production. See Lam (1989) for a discussion.

generations, the intergenerational game heavily favours the elderly (the converse will of course happen if we reverse the meaning of q).

Example 2: Bargaining on the age of retirement

Again let $Q = [0,1]$. Consider two generations t and t-1 and assume that:

$$V(q,\sigma_{t-1}) = (1-q^2)\cdot f^o(\sigma_{t-1})$$

and

$$W(q,\sigma_{t-1}) = (2\cdot q - q^2)\cdot f^y(\sigma_{t-1}).$$

This may have the following interpretation: The three living periods (young, middle, old) of each generation are normalized to have unit length. Then $(1-q)$ denotes the point of time during the old age of generation t-1 when this generation goes into retirement (i.e., $q=0$ means that the old have to work until they die and $q=1$ represents immediate retirement). $f^o(\sigma_{t-1})>0$ denotes the constant instantaneous output the old generation can produce when it is working and the state of the world is σ_{t-1}. On retiring, generation t hands over the control over economic resources to its children's generation t which produces an instantaneous output $f^y(\sigma_{t-1})>0$. Naturally, V decreases in q whereas W increases, both at diminishing rates.[13]

It is an easy exercise to calculate that

$$q^*(\sigma_{t-1}) = \frac{f^y(\sigma_{t-1})}{f^y(\sigma_{t-1}) + f^o(\sigma_{t-1})} \in]0,1[$$

which is decreasing in f^o and increasing in f^y. The total benefit then amounts to

$$W(q^*(\sigma_{t-1}),\sigma_{t-1}) + V(q^*(\sigma_{t-1}),\sigma_{t-1}) = \frac{(f^o(\sigma_{t-1}))^2 + f^o(\sigma_{t-1})\cdot f^y(\sigma_{t-1}) + (f^y(\sigma_{t-1}))^2}{f^y(\sigma_{t-1}) + f^o(\sigma_{t-1})}.$$

This can be employed to determine the outcome of the renegotiation game via Proposition 1. Note that (whatever the initial contract looks like) it will neither happen that the old work until they die nor that they retire immediately.

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13 Check that $V(q,\sigma_{t-1}) = 2\cdot f^o(\sigma_{t-1})\cdot \int_0^{1-q}(1-\tau)d\tau$ which motivates the following interpretation: $(1-\tau)$ is a proxy for the instantaneous productivity of the elderly at time τ. I.e., the productivity decreases with the age of the elderly (think, e.g., of diminishing physical and mental strength). Similarly, check that for the younger generation $W(q,\sigma_{t-1}) = 2\cdot f^y(\sigma_{t-1})\cdot \int_{1-q}^{1}\tau\, d\tau$, which indicates an increase in their productivity with their age (e.g., due to experience gained during their working life).

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time	t		t+1		t+2
gen. t-2	retirement, obtains R_{t-2} from children, benefit V_{t-2}, dies				
gen. t-1	working, contract Γ_{t-1} with children, decides on k_{t-1}, S_{t-1}^2, pays R_{t-2} to parents. benefit W_{t-1}		retirement, obtains R_{t-1} from children, benefit V_{t-1}, dies		
		θ_{t-1} is realized, contract Γ_{t-1} is renegotiated			
gen. t	youth, contract Γ_{t-1} with parents, decides on e_t, S_t^1		working, contract Γ_t with children, decides on k_t, S_t^2, pays R_{t-1} to parents, benefit W_t		retirement, obtains R_t from children and benefit V_t, dies
				θ_t is realized, Γ_t is renegotiated	
gen. t+1			youth, contract Γ_t with parents, decides on e_{t+1}, S_{t+1}^1		working, contract Γ_{t+1} with children, decides on k_{t+1}, S_{t+1}^2, pays R_t to parents, benefit W_{t+1}

FIGURE 1: Time Structure of the Model

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