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Adverse Rybczynski Effects Generated from Scale Diseconomies

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Abstract

We discuss general equilibrium effects of endowment changes in a small open and competitive economy with scale diseconomies. We argue that adverse Rybczynski effects may be observed when inputs are both superior and gross substitutes in the aggregate. The analysis is based on cost functions and allows for more than two goods and factors.

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1 Introduction

Rybczynski's (1955) famous theorem is among the most significant insights of modern pure trade theory. In a nutshell, it says that an increase in one factor's endowment will cause more output to be produced in some sector(s) while, at the same time, production will be lowered in some other sector(s). More precisely, given the standard Heckscher-Ohlin framework of two goods and factors, an industry will always benefit from the extra factor supply provided that factor usage is more intensive in this industry than it is in the alternative one. The latter industry will have to face output losses.

Proofs of the theorem usually rely on the convenient simplification that scale effects can be assumed away due to linearly homogeneous production functions. In particular, it has been known for some time that under variable returns to scale further assumptions need to be made in order to retain the original findings (cf. Jones (1968), Mayer (1974), and Panagariya (1980)). For example, Ingene and Yu (1991, pp. 464-465) have recently shown, in the context of a two-region model of resource allocation, how Rybczynski's theorem may extend to the case of diseconomies of scale when there is allocational uncertainty.

It is the purpose of our note to establish an opposite result which applies in a systematic way to a broad family of technologies with scale diseconomies, i.e. we provide and interpret a sufficient condition for adverse Rybczynski effects in the sense that *all* sector outputs tend to increase in reaction to positive endowment changes. We thereby deviate from the papers quoted above in two major respects. Firstly, we do not restrict ourselves to the arche-type model of two goods and factors (cf. Ethier (1984)). Secondly, our analysis is carried out in dual space, i.e. we are starting from cost functions and hence need not explicitly refer to a 'production possibility locus' (cf. Herberg and Kemp (1969)).

2 The Model

In our model of a small open and competitive economy we consider $n \geq 2$ profit maximizing sectors each producing $x_i > 0$ ($i = 1, \ldots, n$) units of a single net output from at least one net input. The economy's total number of inputs is $m \geq 2$. There are fixed endowments $\mathbf{v}' = (v_1, \ldots, v_m)$ where $v_j > 0$ for all $j = 1, \ldots, m$. (The prime $'$ denotes transposes.) Output prices $p_i > 0$ $(i = 1, \ldots, n)$ are exogenous and taken from the world markets whereas factor prices $w_j > 0$ $(j = 1, \ldots, m)$ are determined endogenously as we assume that inputs do not cross the national borders. We also assume that the minimum cost in each sector i of producing x_i output units at factor prices $\mathbf{w}' = (w_1, \ldots, w_m)$ can be computed from a differentiable cost function $C^i : \mathbb{R}^m_{++} \times \mathbb{R}_{++} \to \mathbb{R}_{++}$ with the following properties satisfied for all $(\mathbf{w}, x_i) \in \mathbb{R}^m_{++} \times \mathbb{R}_{++}$: Cost is a nondecreasing first-order homogeneous and (weakly) concave function of factor prices $\mathbf{w}$. Marginal cost is positive and increases as output increases (decreasing returns to scale in production). There are no fixed costs.

Now suppose that the economy is in an equilibrium state such that marginal costs equal (finite) output prices in all sectors i: $C^i_{x_i}(\mathbf{w}, x_i) = p_i$. Suppose, too, that all factors are at full employment. Hence, $\sum_{i=1}^n C^i_{w_j}(\mathbf{w}, x_i) = v_j$ for all j, where $C^i_{w_j}$ is short-hand notation for $\partial C^i / \partial w_j$ and thus stands for the quantity demanded of factor j in sector i according to Shephard's Lemma. The comparative-statics effects of a change in endowments can now be obtained from totally differentiating these $n + m$ equilibrium conditions for given output prices p_i. This results in the following matrix system:

$$\mathbf{C}'_{\mathbf{xw}}\, d\mathbf{w} + \mathbf{D}\, d\mathbf{x} = \mathbf{o}\,, \tag{1}$$

$$\mathbf{C}_{\mathbf{ww}}\, d\mathbf{w} + \mathbf{C}_{\mathbf{xw}}\, d\mathbf{x} = d\mathbf{v}\,, \tag{2}$$

introducing as $\mathbf{x}$ and $\mathbf{o}$ the vector of sector outputs and the null vector of length n, respectively, and with matrices $\mathbf{C}_{\mathbf{xw}} := (C^j_{w_i x_j})$, $\mathbf{D} := \mathrm{diag}(C^i_{x_i x_i})$, and $\mathbf{C}_{\mathbf{ww}} := (\sum_{k=1}^n C^k_{w_i w_j})$. (Subscripts attached to C again represent partial derivatives. Matrix rows and columns are referenced by indexes i and j, respectively.) Note that with constant returns to scale assumed, i.e. $\mathbf{D}$ possessing zero elements throughout, equations (1) and (2) are combined as (A9) in Jones and Scheinkman (1977). Also observe that in the present context all entries along the diagonal of $\mathbf{D}$ are positive. Hence, firstly, $\mathbf{D}$ is positive definite and, secondly, the inverse $\mathbf{D}^{-1}$ exists and is likewise positive definite. We thus find from (1) that $d\mathbf{x} = -\mathbf{D}^{-1}\, \mathbf{C}'_{\mathbf{xw}}\, d\mathbf{w}$. Hence, using (2):

$$(\mathbf{C}_{\mathbf{ww}} - \mathbf{C}_{\mathbf{xw}}\, \mathbf{D}^{-1}\, \mathbf{C}'_{\mathbf{xw}})\, d\mathbf{w} = d\mathbf{v}\,. \tag{3}$$

3 Results

We are now prepared to establish three propositions. All are concerned with the change in the endowment of a single factor. This factor may (but does not have to) be considered as being specific to one industry. For example, a 'natural resource' might be only used, among other resources, in an 'energy' sector the output of which is for final consumption. Such a scenario is well-known from the Dutch disease literature which emerged when the declines of the Dutch, British and Norwegian manufacturing sectors in the 1960's, late 70's and early 80's were in part attributed to the Schlochteren natural gas discoveries and to Britain's and Norway's oil finds in the North Sea (see Pfingsten and Wolff (1993) for a more detailed exposition and references to the literature). We start with our first proposition, stating that a larger endowment will bring about a reduction of the related factor price:

PROPOSITION 1: *Suppose that the changes in equilibrium factor prices are unique. Then an increase in the supply of one factor will always reduce this factor's price.*

PROOF: To begin with, notice in (3) that $\mathbf{C_{ww}}$ is by definition equal to the sum of the Slutsky matrices $\mathbf{C^k_{ww}} := (C^k_{w_i w_j})$ of all sectors k and is, hence, a negative semi-definite matrix. Next let $\mathbf{R} := \mathbf{C_{xw}}\,\mathbf{D}^{-1}\,\mathbf{C'_{xw}}$. This matrix is positive semi-definite since for every arbitrary column vector $\mathbf{r}$ with length m there exists another vector $\mathbf{z} := \mathbf{C'_{xw}}\,\mathbf{r}$ (possibly equal to the null vector) such that $\mathbf{r'}\,\mathbf{R}\,\mathbf{r} = \mathbf{r'}\,\mathbf{C_{xw}}\,\mathbf{D}^{-1}\,\mathbf{C'_{xw}}\,\mathbf{r} = \mathbf{z'}\,\mathbf{D}^{-1}\,\mathbf{z} \geq 0$ due to $\mathbf{D}^{-1}$ being positive definite. Therefore, $\mathbf{C_{ww}} - \mathbf{R}$ must be a negative semi-definite matrix. Furthermore, uniqueness of $d\mathbf{w}$ requires that $\mathbf{C_{ww}} - \mathbf{R}$ is regular and thus even strictly negative definite. Consequently, premultiplying both sides of (3) by $d\mathbf{w'}$ gives, letting $dv_1 = \ldots = dv_{m-1} = 0$ and $dv_m > 0$ with no loss of generality:

$$d\mathbf{w'}\,(\mathbf{C_{ww}} - \mathbf{R})\,d\mathbf{w} = d\mathbf{w'}\,d\mathbf{v} = dw_m\,dv_m < 0\,. \tag{4}$$

This proves Proposition 1. ■

Our second proposition applies to all factor prices. We assume that inputs are both gross substitutes and superior *in the aggregate.* We thereby neither exclude from a microeconomic viewpoint that some factors may be specific to one sector nor do we deny the existence of selected technologies with no input substitution possible. Formally speaking, we require that all off-diagonal elements of $\mathbf{C_{ww}}$ and all elements of $\mathbf{C_{xw}}$ are positive. Note that the gross substitutes assumption is vital if seen from the viewpoint of stability of a competitive equilibrium (cf. Hahn (1982)). Also note that inputs are always gross substitutes and superior on the micro level whenever a related cost function is generated from a substitutional and homothetic technology.

Our terminal assumption relates to scale diseconomies. They shall be pronounced in the sense that marginal cost is strongly increasing in each production sector. In particular, all (diagonal) entries of $\mathbf{D}^{-1}$ and all off-diagonal entries of $\mathbf{R}$ shall turn out to be small (two examples are given down below). We can then state and prove:

PROPOSITION 2: *Suppose that the changes in equilibrium factor prices are unique. Further suppose that, in the aggregate, all inputs are superior gross substitutes. At the same time, assume that all off-diagonal elements of* $\mathbf{R}$ *are sufficiently small as compared to the elements of* $\mathbf{C_{ww}}$ *due to significant scale diseconomies. Then all factor prices will drop when the supply of one input is increased.*

PROOF: To begin with, let again $dv_1 = \ldots = dv_{m-1} = 0$ and $dv_m > 0$. Next recall that $\mathbf{C_{ww}} - \mathbf{R}$ is a regular and thus strictly negative definite matrix because of our uniqueness assumption, so all of its eigenvalues will come out real and strictly negative. Now observe that inputs have been assumed aggregate gross substitutes. Consequently, all off-diagonal elements of $\mathbf{C_{ww}}$ will be positive. Furthermore, $\mathbf{C_{ww}} - \mathbf{R}$ will have the same property, provided that all off-diagonal elements of $\mathbf{R}$ are sufficiently small. As a result, $\mathbf{C_{ww}} - \mathbf{R}$ will be Hicksian and hence possess a matrix inverse with negative elements only (cf. Takayama (1974, p. 392-393)). Therefore, by (3):

$$d\mathbf{w} = (\mathbf{C_{ww}} - \mathbf{R})^{-1} d\mathbf{v} < \mathbf{o}\,,$$

which completes our proof of Proposition 2. ■

According to Proposition 2, *all* factor prices must fall in equilibrium when *any* factor endowment is increased (cf. Panagariya (1980, p. 516)). Note that factor prices cannot be cut simultaneously under conditions of constant returns with output prices fixed. The reason for this is that, under constant returns, marginal cost equals average cost in all production sectors while, at the same time, average cost is independent of output quantities. In the presence of scale diseconomies, however, marginal cost pricing can also be made possible by means of output changes (cf. Panagariya (1980, Proposition 5)):

PROPOSITION 3: *Given the assumptions of Proposition 2, each sector will produce more output when more is supplied of one arbitrary input.*

PROOF: Note that all elements of $\mathbf{C}'_{\mathbf{xw}}$ and all diagonal entries to $\mathbf{D}$ assume positive values due to the aggregate superiority of inputs and decreasing returns to scale in production. Furthermore, note that $d\mathbf{w} < \mathbf{o}$ (component-wise) according to Proposition 2. The claim is then immediate from matrix equation (1). ∎

This proposition demonstrates that the Rybczynski theorem does not easily generalize to technologies which exhibit diseconomies of scale. In fact, it turns out for a large class of cases specified in Propositions 2 and 3 that all sectors may increase their output when there is a larger factor supply. In addition, the so-called Dutch disease phenomenon (an economy's manufacturing sector appears to decline after a resource boom) need not (and in the specified cases cannot) occur. This is true irrespective of the natural resource being specific to one sector or not. Finally, growth cannot be immiserizing if measured in terms of changes in GNP.

4 Examples

We conclude with two examples of technologies and cost functions, respectively, which are compatible with our propositions. We present in a first example of $n = m = 2$ goods and factors two cost functions which are locally consistent with Propositions 1 through 3, i.e. apply for a given set of vectors $\mathbf{w}$ and $\mathbf{x}$ (but not necessary for other such vectors). One of the inputs will be sector specific and, hence, there is no input substitution in the other sector. As our second example, we sketch a case of $n, m \geq 2$ outputs and inputs and related cost functions with the following 'global' property: For each vector of factor prices $\mathbf{w}$ there exists a vector of outputs $\mathbf{x}$ (with components chosen sufficiently large) such that the pre-conditions of Propositions 1 through 3 are satisfied.

EXAMPLE 1: Let $n = m = 2$ and consider the cost functions $C^1(w_1, w_2, x_1) = w_1 x_1^{1.1}$ and $C^2(w_1, w_2, x_2) = 10\,(0.5w_1^{-8} + 0.5w_2^{-8})^{-0.125} x_2^{1.2}$. The first of these functions comes from a technology with only one input used while the second is dual to a homothetic technology of the CES type. Suppose that in the economy's equilibrium position $w_1 = 1$, $w_2 = 1.5$ and $x_1 = 0.15$, $x_2 = 1$. Then some lenghty computations (to be obtained from the authors upon request) will give:

$$\mathbf{C_{ww}} = \begin{pmatrix} -3.530 & 2.354 \\ 2.354 & -1.569 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.910 & 12.535 \\ 0 & 0.326 \end{pmatrix}, \quad \mathbf{D}^{-1} = \begin{pmatrix} 1.649 & 0 \\ 0 & 0.384 \end{pmatrix}.$$

Hence,

$$\begin{aligned} &\mathbf{C_{ww}} - \mathbf{R} = \mathbf{C_{ww}} - \mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C}'_{\mathbf{xw}} \\ &= \begin{pmatrix} -3.530 & 2.354 \\ 2.354 & -1.569 \end{pmatrix} - \begin{pmatrix} 61.684 & 1.569 \\ 1.569 & 0.041 \end{pmatrix} = \begin{pmatrix} -65.214 & 0.785 \\ 0.785 & -1.610 \end{pmatrix}, \end{aligned}$$

which is a Hicksian matrix. In passing, note that the elasticity of input substitution implied by the second cost function amounts to as much as 9. (Consult Hasenkamp (1976, pp. 49-50) for the computation of this elasticity from a CES cost function.)

EXAMPLE 2: Let $n, m \geq 2$ and consider an arbitrary production sector i. (In what follows, we will suppress index i for brevity.) Suppose that this sector's substitutional production technology is homothetic such that the associated cost function assumes the form $C(\mathbf{w}, x) = c(\mathbf{w})\,h(x)$ with $h(0) = 0$ and $h_x(\cdot), h_{xx}(\cdot) > 0$. In particular, let $h(x) := \ln a - \ln(a - x)$ for all $0 \leq x < a$ (> 0). Also let d stand for the i-th diagonal element of $\mathbf{D}$. Hence, if $x > 0$, we conclude for sector i:

$$\begin{aligned} C_{\mathbf{ww}} - C_{x\mathbf{w}}\, d^{-1}\, C'_{x\mathbf{w}} &= c_{\mathbf{ww}}\, h - c_{\mathbf{w}} c'_{\mathbf{w}} \frac{h_x h_x}{c h_{xx}} \\ &= (c_{\mathbf{ww}} - c_{\mathbf{w}} c'_{\mathbf{w}} \frac{h_x h_x}{c h h_{xx}})\, h \end{aligned}$$

where $h_x = (a - x)^{-1}$, $h_{xx} = (a - x)^{-2}$ and thus $h_x h_x / h_{xx} = 1$. Therefore, all elements of

$$c_{\mathbf{w}} c'_{\mathbf{w}} \frac{h_x h_x}{c h h_{xx}} = c_{\mathbf{w}} c'_{\mathbf{w}} \frac{1}{ch}$$

(including the diagonal ones) will always appear small in absolute size if compared to the corresponding elements of $c_{\mathbf{ww}}$, provided that x is sufficiently close to a. Now suppose that all cost functions $C^j(\mathbf{w}, x_j)$ $(j \neq i)$ possess similar properties. As a result, $\mathbf{C_{ww}}$ will be dominant in $\mathbf{C_{ww}} - \mathbf{R}$ for large x. (Note that $h(x) = x^\alpha$, $\alpha > 1$, would *not* work in the same way, since in this case $h_x h_x/(h h_{xx}) = \alpha/(\alpha - 1) = \text{const.}$)

References

ETHIER, W.J. (1984), "Higher Dimensional Issues in Trade Theory," in: R.W. JONES and P.B. KENEN (eds.), *Handbook of International Economics I*, North-Holland, Amsterdam, 131-184.

HAHN, F. (1982), "Stability," in: K.J. ARROW and M.D. INTRILIGATOR (eds.), *Handbook of Mathematical Economics II*, North-Holland, Amsterdam, 745-793.

HASENKAMP, G. (1976), *Specification and Estimation of Multiple-Output Production Functions*, Springer, Berlin.

HERBERG, H., and KEMP, M.C. (1969), "Some Implications of Variable Returns to Scale," *Canadian Journal of Economics* **2**, 403-415.

INGENE, C.A., and YU, E.S.H. (1991), "Variable Returns to Scale and Regional Resource Allocation Under Uncertainty," *Journal of Regional Science* **31**, 455-468.

JONES, R.W. (1968), "Variable Returns to Scale in General Equilibrium Theory," *International Economic Review* **9**, 261-272.

JONES, R.W., and SCHEINKMAN, J.A. (1977), "The Relevance of the Two-Sector Production Model in Trade Theory," *Journal of Political Economy* **85**, 909-935.

MAYER, W. (1974), "Variable Returns to Scale in General Equilibrium Theory: A Comment," *International Economic Review* **15**, 225-235.

PANAGARIYA, A. (1980), "Variable Returns to Scale in General Equilibrium Theory Once Again," *Journal of International Economics* **10**, 499-526.

PFINGSTEN, A., and WOLFF, R. (1993), "Endowment Changes in Economic Equilibrium: The Dutch Disease Revisited," in: W.E. DIEWERT, K. SPREMANN and F. STEHLING (eds.), *Mathematical Modeling in Economics*, Springer, Berlin, *forthcoming.*

RYBCZYNSKI, T.M. (1955), "Factor Endowments and Relative Commodity Prices," *Economica* **22**, 336-341.

TAKAYAMA, A. (1974), *Mathematical Economics*, Dryden, Hinsdale.

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