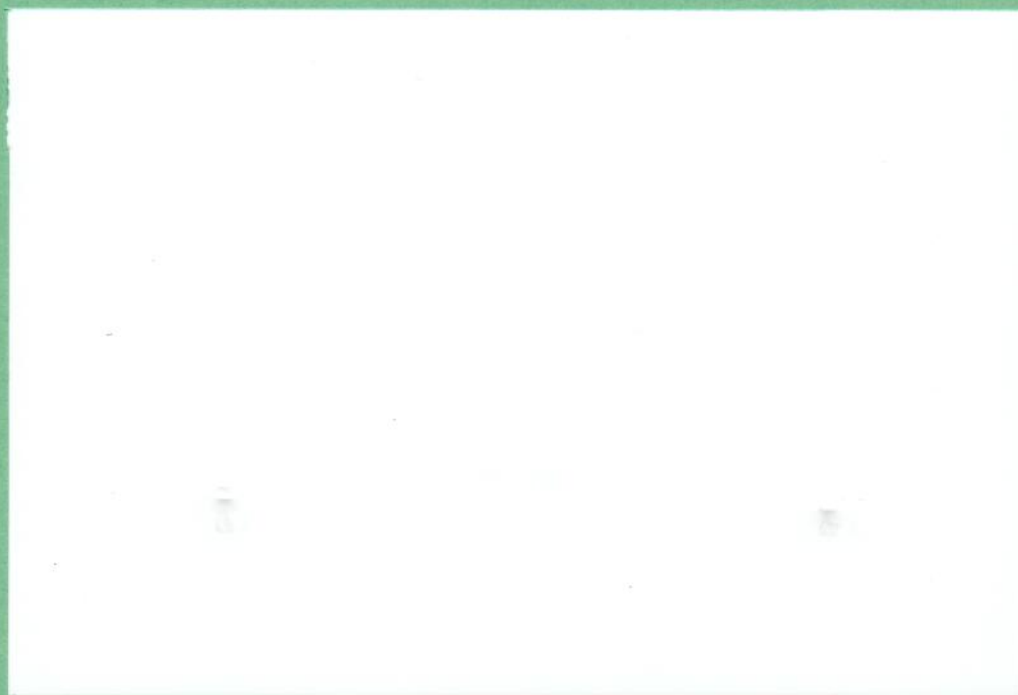


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**CALCULUS OF CONSENT:
A GAME-THEORETIC PERSPECTIVE. COMMENT**

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ABSTRACT:

This is a comment on Urs Schweizer's paper with the above title presented at the International Seminar on the New Institutional Economics in Wallerfangen, May/June 1989. After focusing on problems in selecting rules and outcomes of rules, it discusses the Buchanan-Tullock issue of interdependence costs and optimal majority for the set of classical voting rules. Then some critical remarks are offered on Schweizer's formalisation of the veil of uncertainty, and, finally, it is advocated that cost-sharing rules should be considered as a problem of constitutional choice in its own right.

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Calculus of Consent: A Game-Theoretic Perspective

Comment

by

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With his game-theoretic perspective of Buchanan's and Tullock's "Calculus of Consent" (1962) Schweizer succeeds, in my view, to cast new light on, to provide fresh ideas for, and to generate stimulating novel conjectures and results in the field of constitutional choice. My overall response is, therefore, to welcome his approach as an important step in a direction which seems to be promising for future research. In what follows, I firstly comment on some problems in selecting rules and outcomes of rules which are, in my view, inherent in the game-theoretic approach. Then I turn to the basic issue of optimal majority when choice is restricted to classical rules. My third point relates to the concept of the veil of uncertainty and its formalization in Schweizer's paper. Finally, I offer a few comments on the treatment of cost-sharing rules in the Buchanan-Tullock-Schweizer framework.

1. Problems in selecting rules and outcomes of rules

Selecting institutions or rules is seen as a problem of constitutional choice where each individual evaluates all feasible rules being more or less uncertain about his or her own future role in society. At this constitutional level, the unanimity rule (or the agreement test, in Buchanan's (1984) words) is used to select one particular rule out of the large set of feasible rules which is then to be applied at the operational level. Since the class of potential rules is large, one would like to know whether the choice set can be restricted to a smaller class of rules. It is possible, of course, to exclude all those rules from further con-

sideration, which can be shown to be dominated by some other rule(s) in each individual's ranking of rules. Schweizer conjectures that all non-anonymous rules are dominated in this sense and he provides an example in which a non-anonymous rule is dominated. But it remains open to question how general this result is beyond the suggestive example.

A fundamental theoretical difficulty is that a rule per se does not determine the outcome uniquely but that, on the other hand, individuals cannot evaluate any rule without uniqueness. Clearly Nash equilibria define rational behavior of players in non-cooperative games. Assuming rational behavior means, therefore, that the non-cooperative game associated to each rule (or game form) is predicted to end up with a Nash equilibrium outcome. But since the set of Nash equilibria may be very large, the necessity of "equilibrium selection" or "plausible refinement tests" arises. Certainly, this issue is not specific to Schweizer's analysis but assessing the acceptability of the whole approach requires to evaluate the possibilities of equilibrium selection.

Schweizer argues that if an equilibrium in dominant strategies exists (for an anonymous game) then this equilibrium "might commonly be accepted as the likely outcome" (p. 5), because it allows for a plausible test of refinement. In addition, an equilibrium in dominant strategies can be played in a setting of incomplete information. Let us agree, therefore, that all agents predict the equilibrium in dominant strategies as the outcome. Moreover, suppose also (for a moment) that the prediction of (unique) outcomes is no problem for rules without the dominant strategy property. Does it then follow that dominant strategy rules always outperform the other rules in the agents' ranking (behind a perfect veil of uncertainty)? This is an interesting question worth to be put on the future research agenda. Schweizer conjectures that "...it seems to be the dominant strategy property which establishes the pre-eminence of the classical rules." (p. 10).

The author's Herculean effort of associating a unique outcome to each rule is relatively successful, in my view, as far as anonymous rules with the dominant strategy property are concerned. I

found it both interesting and plausible to extend the revelation principle with respect to anonymity and then proceed with direct mechanisms. However, when it comes to anonymous rules with redistribution at the operational level but without the dominant strategy property, things seem to get very messy. For that case Schweizer provides a complex manual of game transformations and equilibrium selection strategies to be followed by all agents behind the veil of uncertainty. One is tempted to argue that this manual itself raises an issue of "constitutional" agreement on what the proper "rational procedure" should be.

2. Classical rules, interdependence costs and optimal majority

In the first part of his paper, Schweizer does not consider the problem of selecting a rule from the general class of rules (or game forms), but rather focuses on the constitutional choice set of classical rules. His formal approach takes external costs into account while presupposing zero decision making costs, in contrast to Buchanan's and Tullock's analysis. With zero decision costs Buchanan's and Tullock's 'calculus of consent' clearly predicts that the unanimity rule will be selected whereas Schweizer's example implies that "...the rule of unanimity need not be optimal in the absence of decision-making costs". (p. 7). In order to reconcile this "obvious" incompatibility of results without being misled by semantics or disputes about "what Buchanan and Tullock really meant", let me first observe that Schweizer, in fact, disregards the decision making costs in the sense of costs of strategic bargaining i.e. costs arising from efforts to obtain distributional goals. But even under this assumption the costs of collective action are not confined to external costs in the sense of expected costs (imposed on an individual made worse off by the implementation of the project). It is also necessary to look at the expected costs in form of opportunity costs of those beneficial projects that get not sufficient support to be carried through. There is no doubt that Buchanan and Tullock emphasized costs of strategic bargaining as decision making costs. But they also argue that "opportunity costs of bargains that are never made" (Buchanan and Tullock, 1962, p. 69) should be includ-

ed in the bargaining costs, too.

Schweizer's example in Section 2 illuminates the case that the opportunity costs of not realizing a project increase with the probability of a positive net willingness-to-pay. Additional insight can be obtained with an even simpler example: Let "a" and "b" be two positive numbers and assume that each of three persons has the net willingness-to-pay $v_i = -a$ or

m	b = 3 throughout and a = ...	a = 1	a = 2	a = 4	a = 7
1	Eext.cost[1]	0.375	0.750	1.500	2.625
	Eopp.cost[1]	0	0	0	0
	Eu[1]	0.125	0.750	0	-1.125
2	Eext.cost[2]	1.125	0.250	0.500	0.875
	Eopp.cost[2]	0.375	0.375	0.375	0.375
	Eu[2]	1.000	0.875	0.625	0.250
3	Eext.cost[3]	0	0	0	0
	Eopp.cost[3]	1.125	1.125	1.125	1.125
	Eu[3]	0.375	0.375	0.375	0.375

Table 1: Constitutional choice of classical rules

$v_i = b$ ($i = 1, 2, 3$) with probability 0.5. Table 1 gives us the expected payoffs ($Eu(m)$) for the classical rules $m = 1$, $m = 2$, and $m = 3$, when parameter values are $b = 3$ and $a = 1, 2, 4, 7$. It is straightforward that

$$\begin{array}{ll}
 3 \succ 2 \succ 1 & \text{if } a = 7, \\
 2 \succ 3 \succ 1 & \text{if } a = 4, \\
 2 \succ 1 \succ 3 & \text{if } a = 2, \\
 1 \succ 2 \succ 3 & \text{if } a = 1.
 \end{array}$$

Observe that changing the value of the parameter "a" leaves the probability of a positive net willingness-to-pay unaffected. These changes affect, instead, the aggregate net willingness-to-pay in most realisations of preferences $v = (v_1, v_2, v_3)$. There are eight such realisations. If $b = 3$ and $a = 1$, seven of them satisfy $\sum_i v_i > 0$, and in all these cases the project would be adopted under the classical $m = 1$ rule. In contrast, the unanimity rule ($m = 3$) would inefficiently block the project in all cases except for the profile $v = (b, b, b)$, and its expected payoff is therefore low. Table 1 also demonstrates that an individual's expected external costs (Eext.cost) of collective decisions increase, *ceteris paribus*, with decreasing plurality and with increasing utility losses. Conversely, the expected opportunity costs (Eopp.cost) increase with increasing plurality required for adopting the project. Note that the ranking of the classical rules given above for alternative values of parameter "a" can be alternatively calculated with the help of $Eu[m]$ for $m = 1, 2, 3$ such that $j \succ i \Leftrightarrow Eu(j) > Eu(i)$ or with the help of $C(m) := Eext.cost[m] + Eopp.cost[m]$, where $j \succ i \Leftrightarrow C(j) < C(i)$. This observation forms an interesting link to Buchanan's and Tullock's original work reinforcing the suggestion to count as costs not only the utility losses of those persons who are worse off when the project is adopted but also the opportunity costs imposed on those individuals who would have gained from the implementation of a rejected project.

3. On the concept of the veil of uncertainty

Schweizer's formalization of the 'calculus of consent' helped me very much to better understand the original work. But unfortunately, a few doubts about the consistency of the basic structure remained. To be more specific, I assume temporarily that some decision rule has just been adopted for operation in the post-constitutional phase. Then each individual knows his or her precise role or "name" $i \in I_0 = \{1, 2, \dots, n\}$, but he or she does not know what the sequence of projects is that will actually be subject to later collective choices. Since each project is implicitly defined by a profile of individual project evaluations

$(v_1, v_2, \dots, v_n)$ or net willingness-to-pay, uncertainty about future projects can be modelled by assuming that profiles of evaluation are randomly drawn, where the probability distribution of these profiles $f(v_1, v_2, \dots, v_n)$ is commonly known. This is tantamount to saying that only one project is subject to collective choice but the individuals' evaluations for that project are random variables with distribution f . A second type of uncertainty on the operational level is that a player's net willingness-to-pay is strictly private information. This uncertainty causes major difficulties only when we deal with rules lacking the dominant strategy property.

A third type of uncertainty arises at the constitutional level. Here, "...the individual is uncertain as to what his own precise role will be in any one of the whole chain of later collective choices..." (Buchanan and Tullock, 1962, p. 78). Similar, as Schweizer let us denote the set of "constitutional individuals" by $I_c = \{A, B, \dots, \Omega\}$. Clearly, the number of elements in I_c is the same as in I_0 . The decisive question is, however, what the probabilities for a constitutional person, say B , are to be assigned the name 1, 2, ... or n after the constitutional choice has been made. In his discussion of game form (7) Schweizer argues, - correctly, as I think - that the "...veil of uncertainty is the thickest possible if.. [the constitutional persons, R.P.] expect each assignment to be equally likely." (p. 8). On the other hand, he calls the veil of uncertainty perfect if all cumulative distribution functions (4) are equal.

In my interpretation, constraints on the functions (4) are ad hoc constraints on (individual evaluations of) future projects without any impact on the degree of uncertainty of constitutional individuals about their post-constitutional names. It is true that $F_1 = F_2 = F_3$ is sufficient for all individual rankings of rules to coincide, since then it doesn't matter whether a constitutional individual expects to be assigned all names $i \in I_0$ with equal probability. But if the veil of uncertainty is a perfect "veil of ignorance" regarding the assignment of names then the inequality of F_1 , F_2 , and F_3 does not prevent the individual rankings to coincide as Schweizer himself demonstrates on p. 8.

Seit 1989 erschienene Diskussionsbeiträge:

Discussion papers released as of 1989:

- 1-89: Klaus Schöler, Zollwirkungen in einem räumlichen Oligopol
- 2-89 Rüdiger Pethig, Trinkwasser und Gewässergüte. Ein Plädoyer für das Nutzerprinzip in der Wasserwirtschaft
- 3-89 Rüdiger Pethig, Calculus of Consent: A Game-theoretic Perspective. Comment

